DRAFT

GDP growth rate and population

I.O.Kitov

Abstract

It has been found that numerical value of real GDP growth rate in developed countries can be represented by a sum of two terms. The first term is the reciprocal value of the duration of the period of average income growth with work experience, T_{cr}. Current value of T_{cr} in the USA is about 40 years, and in the UK is 35 years. The second term is inherently related to the population change and proportional to the relative change of the number of people of some country specific age (9 years in the USA and UK),

$$(1/2)*dN9(t)/N9(t)$$

, where N9(t) is the number of 9-year-olds at time t. It has been also found that T_{cr} grows as the square root of real GDP per capita.

Hence, the real GDP evolution for a given country is defined by only one parameter - the number of people of the specific age. The population change is the only factor for economic development with the highest efficiency. Prediction for the USA, UK, and France is presented and discussed.

Similar relationship is derived for real GDP per capita. Absolute value of GDP per capita growth in the largest developed countries is also a combination of some economic trend and the specific age population related term. Absolute value of the economic trend term (potential relative growth rate) all the countries is equal to $400 (2002 fixed dollar) divided by the attained value of real GDP per capita. Thus, the economic trend term has an asymptotic value of 0 in all countries.

Inversion of the measured GDP data into population is used to recover corresponding change of the specific age population from 1955 to 2003. The GDP population recovery method is of a higher accuracy than routine censuses.

JEL Classification: J1, E17, O4, C5

Introduction

A comprehensive study of the US personal income distribution (PID) and detailed modelling of some important characteristics of the distributions is carried out by Kitov (2005a). The principal finding is that people as economic agents producing (equivalent - earning) money are distributed in a fixed and hierarchical manner resulting in a very rigid response of the personal income distribution to any external disturbances, including inflation and real economic growth. There is a predefined distribution of relative income (i.e. portion of the total population obtaining a given portion of the total real income) and any place in the distribution is occupied by somebody. A person occupying a given place may propagate to a position with a different income, but the vacant place has to be occupied by the person who was in the new place of the first person. Only exchange of income positions in the PID is possible. This mechanism provides the observed stable personal relative income distribution.

The measured PIDs in the USA corrected for the observed nominal per capita GDP growth rate show a very stable shape during the period from 1994 to 2003. (The US Census Bureau has published PIDs only for this period.) This stability is interpreted as an existence of an almost stable relative income distribution hierarchy in the society, which might be developing very slowly with time. Inflation represents a mechanism compensating disturbance of the PID caused by real economic growth. Inflation eats out of the poor people advantages obtained from the real economic growth.

The economic structure also predefines the observed economic evolution. Only characteristics of age distribution are important for GDP growth rate above some economic trend. The latter is inversely proportional to the attained value of per capita real GDP. Analysis of the two factors of the economic growth is the main goal of this paper. The analysis is focused on decomposition of the real economic growth (GDP) and per capita real economic growth (per capita GDP) in developed economic countries into the two defining parameters.

1. Model for the prediction of GDP growth rate

The per capita GDP growth rate in the USA was used by Kitov (2005a) as an external parameter in order to predict the observed evolution of the PID, its components and derivatives. The PID has been shown to be a simple and predetermined function of this parameter and age structure of the working age population in the USA. The current study interprets this relationship in the reverse direction, however. The observed PID is considered as a result of each and every individual effort to earn (equivalent - to produce) money in the structured economic society as exists in the USA. Thus, the individual money production (earning) aggregated over the US working age population is the inherent driving force of the observed economic development. The working age means the age eligible to receive income, i.e. 15 years of age and above. This effectively includes all retired people.

The principal assumption made by Kitov (2005a) and retained in the current study is that GDP denominated in money is the sum of all the personal incomes of all the people of 15 years of age and above. This statement is not a redundant one formulating the income side of GDP definition. This statement unambiguously defines the upper limit to the total income (Gross Domestic Income) or GDP which can be produced by a population with a given age structure and characterized by some attained value of the GDP per capita. Because the age structure is given and the individual incomes in the society are defined by a strict relationship between age and per capita GDP (Kitov, 2005a), the total potential income growth has to be also predefined.

A person produces exactly the same amount of money as s/he receives as income. This formulates not only global balance of income (earnings) and production, but a more strict and important local balance. The structure of a developed economic society confines its possible evolution because everybody has its place (position in PID) and produces according to this place.

This approach also implies that there is no economic means to disturb the economic structure of the society. For example, it is impossible to reduce poverty or to limit individual incomes of rich people compared to the level predefined by the economic structure. According to the PID observations in the USA, positions of the poor and rich people in the structure (as defined by PID) have to be occupied. The extent to which the all positions are occupied may be linked to the degree of economic performance. Performance in a disturbed income structure would be reduced compared to its potential level only defined by the per capita GDP and the age structure of the population. Thus, only some non-economic means are available to fight poverty, i.e. society can provide higher living standard, but not higher incomes. When applied, any economic means (income redistribution in favor of poorer people) have to result in economic underperformance.

Per capita real (or nominal) GDP growth rate is uniquely determined by the current distribution of the personal income which, in turn, depends on the population age distribution. As shown by Kitov (2005b), the mean personal income distribution is only governed by values at two points - the starting point of the distribution and T_{cr}, i.e. the value of work experience characterized by the highest mean income. The integral of the product of the mean personal income and the number of people with given work experience over the work experience range gives a GDP estimate. Thus, one can suppose that numerical value of real GDP growth rate in developed countries, which are characterized by a stable economic structure or PID, can be represented as a sum of two terms. The first term is the reciprocal value of the T_{cr} which is often called the economic trend or potential. Current value (2004) of T_{cr} in the USA is of 40 years, i.e. the current economic trend (including working age population growth) is 0.025. (In the second part of the paper we will consider a case of per capita GDP economic trend). The second term is inherently related to the number of people of some specific age. This defining age has to be determined by some match study and varies with country. In the USA and the UK, the age is 9 years. In European countries and Japan it reaches 18 years. This term creates the observed high frequency fluctuation in the GDP growth rate and is expressed by the following relationship $0.5*dN(t))/N(t)dt$, where $N(t)$ is the number of people with the specific age at time t. Thus, one can write the following relationship for the GDP change:

$$g(t)=dG(t)/G(t)dt=0.5dN(t)/N(t)dt+1/T_{cr}(t) \quad \textbf{(1)}$$

where $g(t)$ is the real GDP relative growth rate, G(t) is the real GDP as a function of time.

Completing the system of equations is the relationship between the growth rate of per capita real GDP and T_{cr}:

$$T_{cr}(t)=T_{cr}(t_0)\ sqrt(\int(1+g-n)dt) \quad \textbf{(2)}$$

where $n(t)=dNT(t)/NT(t)dt$ is the time derivative of the total working age population relative change during the same period of time. The term in brackets under integral is the per capita real GDP growth rate. So, the critical time evolves as the square root of the total real per capita GDP relative growth during some given period from the starting time t_0 time to time t.

2. GDP growth rate prediction

Using equations (1) and (2) one can predict the observed evolution of real GDP in the USA. The defining age of 9 years in the USA is found by matching the predicted and observed GDP evolution. Consequent substitution of various single age distributions in (1) gives the best fit for nine years of age. The single year population estimates used in the study are available at the U.S. Census Bureau web-site [2004a – 2004c]. There are several different sets of the population data which undergo modification as new information or methodology becomes available. Censuses are carried out every ten years and the last was accomplished in 2000. After the 2000 census, all the population estimates for the period from 1990 to 2000 were adjusted in a way to fit the new census counts (US Census Bureau, 2005d). The difference between the estimated and counted population at April 1, 2000 is called "the closure error". The population estimates made for this period were based on the 1990 census and the measured population components changes during this decade. These estimates showed sometimes very poor results compared to the counting. Thus, there are two data sets for the period called below intercensal and postcensal following the definition by the U.S. Census Bureau. For the period before 1990 only one data set is available.

Figures 1a and 1b present the measured (BEA, 2005) and predicted by (1) and (2) real GDP growth rate values. The number of 9-year-olds is taken from the intercensal and postcensal estimates respectively (US Census Bureau, 2005d). Overall, the observed and derived time series are not in a good agreement. A relatively good agreement is observed only near the years of 1980 and 2000.

The used single year of age population estimate is adjusted to the decennial censuses. So, the time series has to provide the best estimates. There are some doubts, however, that the data are over-smoothed and corrected in a way to suppress any possible large change in the single year population. As stated in the overview of the population estimates (US Census Bureau, 2005) - "These [population] estimates are used in federal funding allocations, as denominators for vital rates and per capita time series, as survey controls, and in monitoring recent demographic changes. ". This goal is slightly different from that of accurate prediction of the number of nine year old children need for the GDP prediction.

The single year population estimates are based on decennial censuses. One can expect that the censuses population distributions are not strongly biased by corrections, adjustments and modifications to the extent the post- and intercensal estimates are. The single year of age distributions can be projected back and forth in time in order to evaluate the number of the 9-year-olds before and after the census. For example, the number of nine-year-olds in the next year from the census is considered as equal to the number of the eight-years-olds in the census year, and so on. The larger time gap between the census and the predicted year the larger is the error induced by all the demographic changes during these years. The same procedure of projection of the single year population can be applied to any annual distribution.

Because we use not absolute but relative population changes in adjacent years, the bias might be not so large. For example, one can consider a process of 1% population growth per year. This value is close to the total population change observed in the USA for the last 40 years. If every single year of age population grows at the same rate, the ratio of the adjacent single year populations is not affected by the population growth. If the adjacent single-year populations $a(n)$ and $a(n+1)$ (of age n and $n+1$) undergo a constant absolute growth by a number of people p every year, then the corresponding ratio will be of $a(n+m)/a(n+1+m) \approx 1+r(1-mp/a(n))$ in m years, where r is the initial ratio. If the absolute growth $mp<<a$ then the approximation gives a relatively high accuracy. If the absolute change is not synchronized between adjacent single years of age cohorts, the ratio may be disturbed.

Thus, one can expect a much smaller relative change between adjacent cohorts than absolute one. The degree of the relative change is not well determined, however, because of relatively low accuracy of the measurements used in the population estimates. The accuracy may be evaluated by a comparison of the postcensal estimates and decennial censuses. Difference between the predicted from continuous observations and counted numbers is as high as several percent and is particularly high for the ages between 5 and 10 (West *et al.,* 1997).

Estimates of the United States population currently are derived quarterly by updating the resident population enumerated in Censuses through the components of population change. The following components are used: census enumeration of resident population+ births to U.S. resident women - deaths to U.S. residents + net international migration + net movement of U.S. Armed Forces. This estimate is accomplished for every single year of age, race and gender. The former is of the importance for the study.

Figures 2 through 5 compare the real GDP measurements and the GDP growth rate predicted from the 9-year-old population estimates based on consequent censuses – from 1980 to 2000. Figure 2 illustrates how well one can predict the real GDP growth rate around 1980. The predicted and observed curves are very close and have similar oscillations, but of slightly different amplitude. The curve predicted from the 1980 population estimate (census year estimate) is much closer to the real GDP growth rate curve than the curved obtained from the 9-year-olds estimates presented in Figure 1. This discrepancy is consistent between the curves based on particular censuses and on 9-year-olds estimates obtained with later corrections and modifications.

Figure 3 demonstrates predictive power of the 1990 census. The 9-year-olds estimate (see Figure 1) for the period around 1990 shows very poor correlation between the observed and predicted GDP growth rate. One can conclude that the curves can not represent the same process due to a substantial difference in amplitude and shape. This is quite opposite to the prediction based on the 1990 population estimate. The amplitude and shape of the predicted GDP is almost the same as those observed near 1990. The most important observation is that the census population estimate unambiguously indicates the observed recession in 1991, i.e. one year before the drop of economic activity. Moreover, the drop could be predicted several years before if the population estimates between 1982 and 1991 would be accurate enough. This kind prediction is of a high interest for the business because it allows foreseeing future inevitable recessions.

The same is valid for the 2000 population estimates. The Census Bureau gives two estimates for the period between 1990 and 2000: postcensal and intercensal. Figures 4 and 5 display the GDP prediction based on the intercensal and postcensal population estimates respectively. The intercensal estimate gives a good timing of the 2001 recession, but underestimates amplitude of the drop. The postcensal estimate gives the same timing but overestimates the amplitude. Both estimates also give a very good timing and relatively accurate amplitude for the two previous recessions. Thus, at least three previous recession periods are well described by the 9-year-old population change as described by relationship (1) if to take the original age distributions.

The intercensal estimate represents a corrected version of the postcensal one and considersnot only continuous data on birth and depth rate, and international migration, but also the final distribution obtained in the 2000 enumeration. One can find a typical trend in the intercensal estimate which consistently smoothing the population changes between subsequent years of age. Figure 6 demonstrates this observation comparing the two predicted GDP time series. There is no actual distribution, however, which can be used to estimate absolute accuracy of the postcensal or the intercensal estimates, i.e. the difference between the actual and enumerated numbers. One can only estimate relative changes made by the Census Bureau and compare those to the observed discrepancy between the real GDP growth rate estimate and the predicted from the 9-year-olds estimates.

One should also bear in mind that the GDP estimates are also exposed to corrections induced by late data and methodology changes. The corrections may be as high as 1 percentage point and be extended 10 years back in past. For example, the US Bureau of Economic Analysis published in July 2005 some corrections of GDP estimates for previous years that considerably changed the GDP values between 2001 and 2004. Presumably, GDP estimates are only 0.5 percentage point accurate and any discrepancy of such an amplitude between the observed and predicted GDP values might be induced by the real GDP estimate uncertainty and not only population miscount. The latter, however, has much lower accuracy as discussed below.

3. Accuracy of the population estimates

Accuracy of the population estimates used in the GDP prediction is of a crucial importance for the study. The prediction of the recession periods between 1980 and 2004 in time and amplitude is possible due to the fact that the observed changes were large enough and can not be smoothed even by the Census Bureau corrections. Between the census years, however, one would like to have a better agreement between the observations and predictions.

There are two questions in the study related to the accuracy of the estimates:

- Can one recover the actual number of 9-year-olds from the Census Bureau population estimates?

If not,

- Does the 9-year-olds population, which accurately predicts the observed real GDP growth rate, contradict the Census Bureau estimates?

There are two general sources of error for the population estimates. The first is the census miscount induced by many reasons. The postcensal estimates of accuracy undoubtedly show that the accuracy of the enumeration depends on age. The largest miscount corresponds usually to the age range between 5 and 10 years. The highest accuracy of the enumeration is observed in the age range between 20 and 30 years. The miscount of children is an underestimation as a rule. It can reach 5 percentage points.

Figures 7 and 8 demonstrate the changes made by the Census Bureau to the single year of age populations. Figure 7 displays evolution of the postcensal estimates for 7-, 8-, 9-, and 10-year-olds. The curves are shifted forward by a number of years equal to the difference of the ages (for example the curve for 7-year-olds shifted by 2=9-7 years forward) in order to trace the same cohort by single year of age. The order on the curves looks suspicious because the number of 8-year-olds is by almost 300,000 less than the number of 9-year-olds in the next year and 150,000 less than the number of 7-year-olds in the previous year. This means that the number of people in a cohort by single year of age initially drops by 150,000 (from 7 to 8 years of age) and then increases by 300,000 in the next year. To improve this strange behavior the Census Bureau has made considerable correction of the numbers between 1990 and 2000 as shown in Figure 8. The difference between 7-year-olds and 8-year-olds is gradually decreased from 150,000 in 1990 to almost zero in 2000 and then the 8-year-olds curve goes above the 7-year-olds curve. Both curves are approaching the 9-year-olds curve between 1990 and 2000. The procedure for the correction implied proportional decrease of the population difference, i.e. a constant rate of catch up. This means that no possible changes induced by the changes in birth and depth rates, and international migration is taken into account in the correction process. The author concludes that there is no possibility to recover some actual 9-year-olds time series from the US Census Bureau data.

4. Population estimates by the real GDP evolution

One can easily find the 9-year-old population corresponding to the observed GDP growth rate. The number is described by the following relationship which represents an inversed version of (1):

$$d(lnN(t)) =2(g-1/Tcr))dt \quad \textbf{(3)}$$

The number of 9-year-olds in a given year *i* is equal to the number in previous year *i*-1 times the term, which reflects the difference between the observed real GDP growth rate, $g(t)$, and the potential growth rate corresponding to the observed real GDP per capita value as defined by relationship (2). This working experience or critical time T_{cr}, depends only on the real GDP per capita. Thus, the term in brackets is completely defined by the real GDP. It is worth noting that T_{cr} can be determined independently from the personal income distribution reports of the Census Bureau. The term in brackets is very sensitive to the evolution of T_{cr} in time. If the observed long-term behaviour of GDP is not accurately described by the term $1/T_{cr}$, discrepancy between the observed and predicted number of 9-year-olds has to be large.

The only value defining behaviour of $N(t)$, considering $g(t)$ is obtained from an independent set of observations, is its initial value for some given year, $N(t_0)$. One can vary this value in order to reach the best fit between the Census Bureau 9-year-olds population estimates and the value predicted by (3). In the study, some year between 1950 and 1960 was selected. The principal goal of the fitting process was to reach the best agreement during last 10 to 30 years. The period before 1970 is characterized by a lower accuracy of the population estimates whether made by the Census Bureau or projected from the population age pyramid for some specific years.

Figures 11 through 16 present a series of comparisons between the estimated and predicted number of 9-year-olds for years between 1960 and 2001. The limits of the population estimates uncertainty adopted in the study is of 5%. Every comparison has its specific target period. Thus, there are several predictions corresponding to various initial values $N(t_0)$.

Figure 11 shows results of the prediction for the 1980 population estimate. The number of 9-year-olds is obtained by the back projection procedure as described above. The target period for the prediction is between 1970 and 1990 with a center at 1980. One can observe an almost precise prediction near 1980. Before 1975 and after 1985, the curves diverge. For the period before 1975 this behaviour can be explained by a growing with time number of people for the same cohort by single year. Obviously, due to a positive international migration into the USA, the number of 9-year-olds in 1960 was lower than the number of 29-year-olds in 1980. Following the same logic, one can expect a larger number of 9-year-olds than the predicted one by the projection of younger population, as observed.

Figure 12 compares 1990 population estimates. The years around 1990 are described not so successfully as those around 1980 and 2000. There is a notable oscillation in the estimated number, however, that mimics to some extent the predicted behaviour. The worst period for the prediction is between 1984 and 1988, where the discrepancy is the largest and the predicted curve even goes out of the 5% uncertainty bounds. The 1990 population estimate is very well matched by the predicted curve for the period before 1980. Despite an obvious underestimate of the population level the predicted curve gives an excellent description for the shape of the estimated curve. Also the predicted curve shows an excellent agreement with the observed one towards 2000.

Figure 13 and 14 present similar comparison of the counted and predicted from the observed GDP number of 9-year-olds. The counted number is obtained as a projection of the 2000 population estimates – post- and intercensal respectively. The predicted curve almost coincides with the postcensal population estimate during the period between 1996 and 2001. The intercensal estimate which is corrected for the 2000 census enumeration also provides a good approximation for this period, but does not contain a small peak near 2002 and is characterized by a steeper drop of the 9-year-olds number after 2001.

Figure 15 illustrates a comparison of the predicted curve with the birth rate in the USA shifted by 9 years ahead. The birth rate curve, however, is also prone to modification by migration during the 9 years. General features of the birth rate curve are well predicted and only the observed population level is higher than predicted.

Figure 16 compares the predicted and observed 9-year old population. The population estimates (post- and intercensal) resulted from consequent modifications and updates of the decennial censuses and intercensal population estimates, and include a number of corrections related to some independent methods of population estimates. This is the result of best efforts of the Census Bureau to present a consistent population universe which tries to balance all available information and theoretical knowledge. Hence, one can assume that the requirement of a balanced population universe contradicts accuracy of the single year population estimates especially during first decade of life. Nevertheless, the 9-year old population predicted from the observed real GDP describes well the estimated population of this age.

Concluding this section we summarize our findings. The best agreement between the estimated and predicted number of 9-year old population is observed for the projected decennial census population from 3 to 5 year around the year of enumeration. Migration processes do not disturb the population age distribution much. In the years between the censuses, the curves diverge the most. The population estimates for these years are obtained by a component method relied on independent measurements of the birth and death rate and migration dynamics. All these components might be inaccurately measured and/or biased. The methods for correction of the single year of age population distribution are very likely to be inaccurate as follows from the comparison of the postcensal and intercensal values. The methods do not consider any possible fluctuation of a single year population between the census years and just distribute proportionally "the closure error" over the years. One can assume that any curve with the same values at the census years can fit the observations, including the curve which is obtained from the real GDP observations according to relationship (3). This is the principal conclusion an the answer to the second question of section 3 – the GDP predicted curve does not contradict the observed population estimates considering their accuracy and methods of correction and modification.

The predicted curve contains two principal components – a high frequency one related to the 9-year old population change and a trend component. The latter is consistently subtracted from the observed GDP growth rate. The component depends on the evolution of the measurable by independent methods value of the time necessary for an average person to reach his/her peak mean income – T_{cr}. Thus, the agreement between the predicted and observed population supports the hypothesis of the dependence of the real GDP evolution on two principal parameters. The better one knows the 9-year-old population distribution, the more accurate is the prediction of real GDP one can obtain. Theoretically, it is possible to measure the population with no uncertainty, i.e. precisely. In turn, the GDP growth rate prediction will be as precise as the population measurement.

There is no estimate of the absolute accuracy of the GDP measurements. The BEA provides only estimates of relative accuracy, i.e. potential changes of the GDP measured values according to later updates induced by obtaining more accurate and complete data and modification of calculation procedures.

5. The United Kingdom and France

For an economic theory it is important to be a universal one and applicable to similar objects. There are several large developed countries providing information on population age distribution and real GDP growth rate. Prediction of the GDP from population and the reversed procedure is accomplished for these countries as well. Here we present result for the UK and France. Figures 17 through 25 present results of the analysis in the same order and by the same methodology as for the USA. The general conclusion is that the observations do not contradict the theoretical approach developed for the USA. One can observe very good correspondence between the deep recessions in the beginning of 1980s and 1990s and the lack of 9-year-olds in the UK. Same is valid for France near 1993. The only principal difference between the UK and France is that the defining age for France is 18 years. This age occurred to be the defining one also for Japan, Germany, Austria, and Russia – the countries studied by the author who has no explanation of the age difference. Accuracy of the population estimates for the UK and France is hard to estimate, but some features unveil some artificial character of the population age pyramid construction.

6. Per capita GDP

In previous sections we have developed a model explaining the observed real GDP growth rate variations in past. We have linked the GDP observations with two principal parameters – duration of the mean income growth period and the number of 9-year-olds in the USA. There is no explanation of these relationships. The model is an empirical one and do not pretend to explain individual and/or social behavior resulting in increase of the period of the mean income growth with increasing per capita GDP. Economic life of a country is a complicated system of interactions between people, which lead to a hierarchical and rigid structure of personal income distribution as found by Kitov (2005a). It was shown that any change in population age structure and real economic growth induce predefined changes in the PID, i.e. there is a functional dependence between these parameters. They create a dynamic economic system similar to those observed in natural sciences.

There is another parameter characterized by a potential predictability with the evolution of an economy. This parameter is an absolute value of per capita GDP growth through time. One can again distinguish two principal sources of per capita GDP growth – the change of 9-year old population and the economic trend related to per capita values. The trend has the simplest form – no change of absolute growth values. This is expressed by the following relationship:

$$dG/dt=A \quad (4)$$

where G is the absolute value of GDP per capita, A is a constant depending on country. Solution of this equation is as follows:

$$G(t)=At+B \quad (5)$$

where $B=G(t_0)$, t_0 is the starting time of the studied period. Relative growth rate is then expressed by the following relationship:

$$dG/Gdt=A/G \quad (6)$$

Relationship (6) indicates that the relative growth rate of the per capita GDP is inversely proportional to the GDP per capita, i.e. the observed growth rate should decay in time with an asymptotic value equal to 0.

Figure 26 presents an approximation of the observed GDP per capita absolute growth values by a constant for the USA ($485 – 2002 US dollars), France ($405), Italy ($405), Japan ($480), and the UK ($378). The constant is obtained as a mean value of the absolute growth 1950 through 2004, i.e. 55 years. For the USA, a longer period is also presented with its own estimate of the growth constant. One can see that the values of the mean growth are between $400 and $500. Only the USA and Japan have a larger value among the countries with the largest economies. Germany is not present due to a change the country underwent in 1990s. (No continuous GDP per capita estimate is available for the country.) Smaller economies also are not analyzed because their economic behavior might be biased by some factor of outer origin. For example, presence of some large neighboring economies making the production of the small country oriented mostly to the demand of the large-size neighbor.

Another possibility is a development of a monopoly service or production driving the economy for a short or longer time period. In all these cases, the small economy can be considered as an industrial or service part of a bigger economy and can not be sustainable by itself. An analog of a small economy can be an industrial branch of a large country that usually develops at a rate different from all other industry branches, i.e. increasing or decreasing its part in the total production, but can not be independent. Thus, economic growth of a small economy can not be described by the same relationship as that of a big economy, where effects of fast increase in one sector of economy are usually compensated by decrease in another sector.

When applied, linear regression (equation for the linear regression is given for the countries in Fig. 26) gives practically the same constant line for France, Italy, and Japan, i.e. the best approximation of the linear growth is just a constant for these countries. For the USA and UK, linear regression indicates some positive trend of the GDP per capita absolute growth with increasing GDP per capita. As shown later, this increase is related to the effect of the population component change. One can predict for the USA a downward trend in future, when and if the effect of population component of growth will disappear. Such a behavior was clearly observed in Japan in 1960s and 1970s, when the population effect was very strong and the per capita GDP absolute growth increased. The effect has being compensated during the last 20 years due to a severe drop in birth rate. One can expect the trend to continue in Japan because of the negative influence of the population change. If the constant of the absolute growth in Japan is the same as in other four countries, i.e. ~$400 as shown below, then the accumulated population effect is of $80. In the UK, the observed increase of the absolute per capita GDP growth in recent years might be a compensation for the negative population effect in past.

Figure 27 summarizes GDP mean absolute growth for the developed countries. One has to bear in mind, however, that the mean values include input from both sources of growth. So, the mean value might not represent the absolute growth constant. The population change factor (relationship (1)) should be subtracted from the total absolute growth value. For example, the 9-year old population in the USA changed from 24022326 in 1950 to 4173171 in 2002. The total input of the population change in the observed economic growth is 0.5*(4173171-2402326)/ 2402326=0.37. Per capita GDP grew during this period from $12123 to $38345 or by 2.16 times. The total growth can be split into the population related component of 0.37 times and the economic trend component of 2.16-0.37=1.79 times. The mean GDP per capita growth was $485 and includes $401=$485*(1.79/2.16) of the constant absolute growth (economic trend) and $84=$485*(0.37/2.16) of the population related growth. For France this effect is much weaker than for the USA – the number of 18-year-olds changed from 649011 in 1950 to 801095 in 2001, i.e. the population effect of the observed growth is only 0.12 or 3 times lower than in the USA. The total growth in France was from $7009 to $28956 or 4.13 times ($406 per year in average) from which only 100*(0.12/4.13)=2.9% were related to the specific age population change. When corrected for the population change, the observed mean per capita GDP growth in France is almost the same as in the USA: $394=0.97*$406. The same procedure can by applied to other countries when corresponding data is available.

Another principal correction has also to be applied to the per capita GDP values. This is the correction for the difference between the total population and population of 15 years of age and above as discussed by Kitov (2005a). Only this economically active population should be counted than per capita values are calculated. Relevant data is not available for the countries except the USA and France. We use the published (uncorrected) per capita GDP values bearing in mind potential bias induced by the discussed population effects. For the USA and France all above mentioned corrections are applied during the analysis.

Figure 28 illustrates per capita GDP relative growth rate as defined by relationship (6). The observed values are shown together with some economic trend curves obtained by a power law regression by a function A/G (where A is the absolute value of constant growth; G is the GDP per capita). Black and red curves correspond to the mean absolute growth values for the period 1930 through 2002 (A=$399) and 1950 through 2002 (A=$535), respectively. The regression is conducted not by the least squares because this standard regression does not preserve the total observed growth during the period, i.e. the sum of all growth rate values for the observed GDP values. We used a regression which minimizes the overall deviation of the observed relative growth values from the best-fit function A/G. Such a regression preserves the total observed growth for the given period with the observed values of per capita GDP. The relative growth rate is a poor parameter to describe economic growth for a relatively long time interval. The relative growth rate is calculated with changing base value. One can not even sum (or multiply) the relative growth rates in order to obtain some reasonable value of the total growth. For example, an averaged over 50 year long interval relative growth rate of 2% (i.e. mean value of 50 independent measurements) gives a lower absolute growth than an averaged rate of 1.9% if the amplitude of oscillations around the averaged value in the first case is 5 time larger than in the second. Figure 29 illustrates this fact - two curves correspond to oscillation amplitude of 0.05 around the mean growth rate of 1.02 and of 0.01 around the mean growth rate of 1.019 during 50 years. Formally, the average growth rate of 1.02 results in lower total growth of 2.53 than that for the average growth rate of 1.019 reaching 2.56 at the end of the period. Thus averaging of relative growth rates over long intervals is not a correct procedure because it depends on the growth rate oscillation amplitude. Absolute growth characterizes the evolution of per capita GDP in an appropriate way because the averaging operation is correct.

The magenta curve in Fig. 28 illustrates the relative growth related to the economic trend only, i.e. to the total economic growth less the population component. As found above, this value is approximately 1.79/2.16=0.83 of the observed mean absolute growth for the period 1950 through 2002. Oscillation of the growth rate values around this curve has to be explained by the population factor. Here and below all the per capita GDP values are corrected for the difference between the total population and the population of 15 years old and above. The ratio of the populations in the USA changed from of 1.37 in 1950 to 1.27 in 2002. In 1930, the ratio was 1.41. Thus the per capita GDP values are larger by 1.37 to 1.27 times than in the original table.

One can redraw equations (1) and (3) in order to replace real GDP values with per capita GDP values and also incorporate economic trend values defined by relationship (6):

$$g_{pc}(t)=0.5dN(t)/N(t)dt+A/G(t) \qquad \textbf{(7)}$$

where $g_{pc}(t)$ is the per capita GDP relative growth rate, $G(t)$ is the real per capita GDP as a function of time. Constant A is

$$d(lnN(t))=2(g_{pc}-A/G(t))dt \qquad \textbf{(8)}$$

Usage of relationships (7) and (8) is similar to that of relationships (1) and (3). Figure 30 presents results of the per capita GDP growth rate prediction by using 2000 postcensal population estimate according to relationship (7). As previously, the number of 9-year-olds was derived by projection of the age structure back and forth in time. As in the case of the total GDP prediction, principal features of the observed curve are in a good agreement with the predicted ones, including severe drops of per capita GDP during the years of recession. There is no possibility to reach a better agreement because of the limited accuracy of the population estimates compared to that for the per capita GDP.

Figures 31 through 34 display results of the prediction of the number of 9-year-olds from the observed per capita GDP according to equation (8). The figures are similar to those for the total real

GDP. The best fit is obtained for the number of 9-year-olds as estimated by the Census Bureau in the postcensal estimation procedure. The curves almost coincide between 1960 and 2002. As discussed before, the estimated curve is slightly over smoothed compared to the census population estimates.

Hence, the observed behavior of the per capita GDP growth rate can be successfully described by the proposed model of two sources of growth – inherent growth of an economy usually called economic trend and the change of the specific age population. If to replace the estimated population (red curve) by the predicted population (blue curve) one can reach absolute prediction accuracy. It is worth noting that the specific age population change has a period of 30 years. Accordingly, the GDP growth rate reveals the same feature merged with the decreasing economic trend. Oscillations with shorter period are also clear. All in all, the presented model for the economic growth does not contradict observations and has a predictive power that only depends on the capability to accurately count people of some given age.

Results obtained by the same procedure applied to France are presented in Figure 35. This comparison is similar to that presented in Figure 25 because the influence of the population component is just minor according to our estimates of the total change of the number of 18-year olds from 1950 to 2002. In the long run, the per capita GDP and total GDP depend only on inherent growth of the French economy.

7. Discussion and conclusions

We have presented an empirical model predicting real GDP behavior by using only two parameters – per capita GDP and number of people of some predefined age. There is an important difference, however, in economic trend for the total real GDP and per capita GDP. The latter is related to the (observed) constant absolute growth and corresponding relative per capita GDP growth rate inversely proportional to the per capita GDP itself. The former is related to the (observed) evolution of some critical work experience, T_{cr}, inversely proportional to the square root of the per capita GDP. In the frame of this study, one can not decide which of the two parameters is defining – value of the absolute growth or evolution of T_{cr}. To distinguish between the two effects one need more data on T_{cr} evolution in different countries. Formally, it is possible to obtain the observed total GDP economic trend inversely proportional to the square root of the per capita GDP if to add the observed change of the total working population to the predicted change of the per capita GDP inversely proportional to the per capita GDP.

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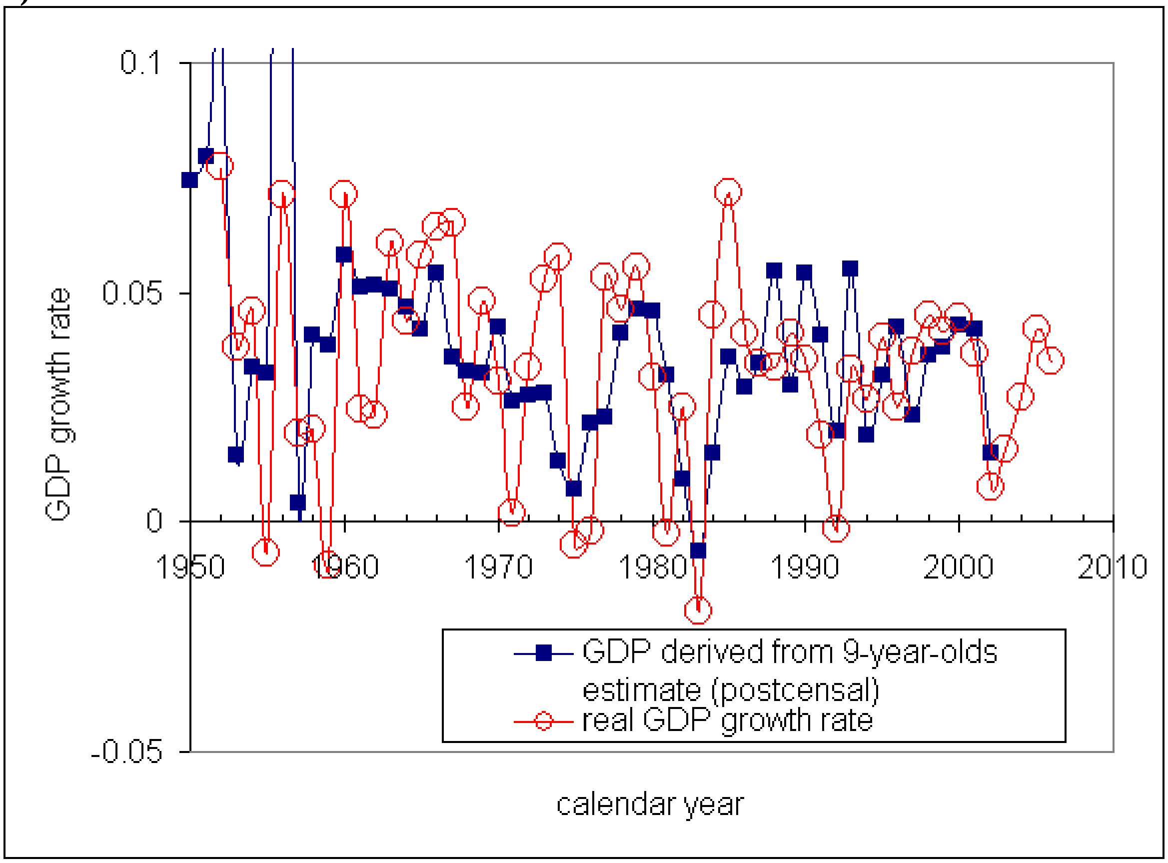
Figures

a)



0.1
0.05
0
-0.05
GDP growth rate
1950
1960
1970
1980
1990
2000
2010
GDP derived from 9-year-olds estimate (postcensal)
real GDP growth rate
calendar year

b)

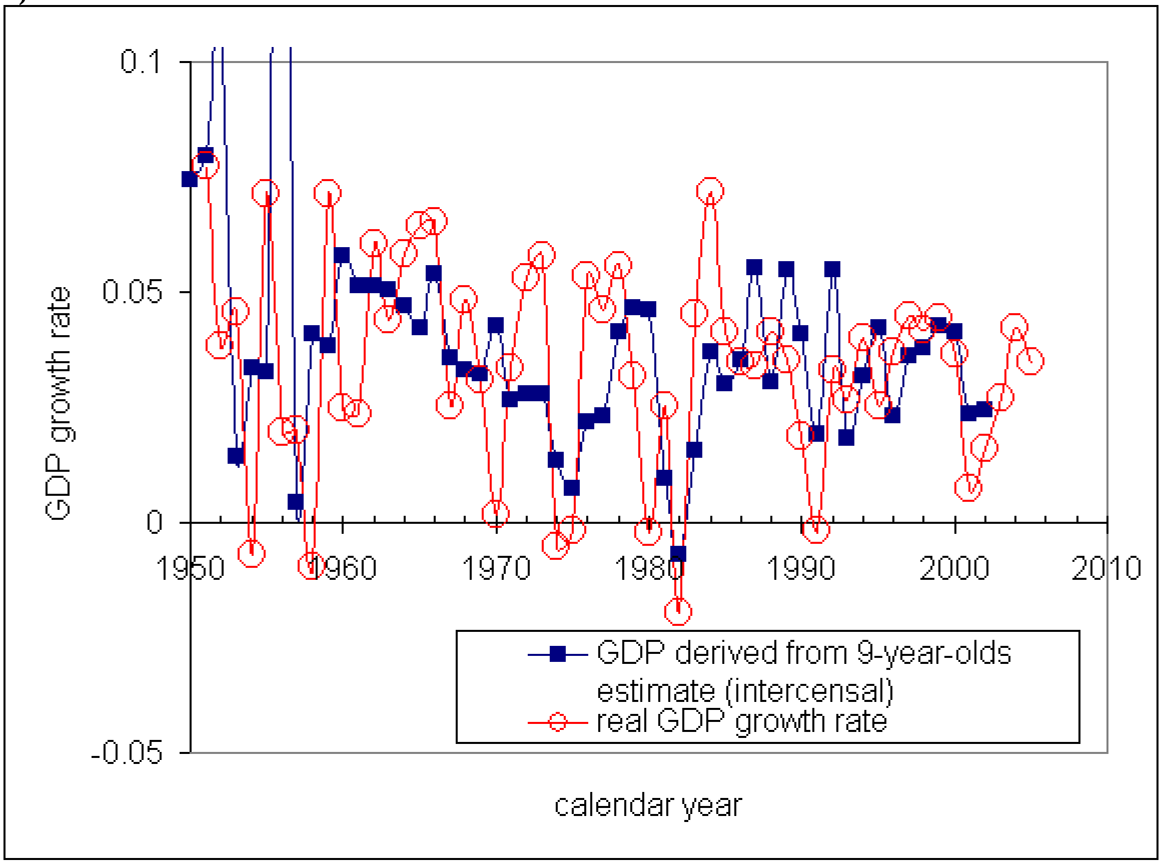


Fig. 1. Real GDP growth rate in the USA for the period between 1950 and 2004. Comparison of the measured values and values predicted by the single year of age population estimates (9-year-olds): a)2000 postcensal estimates; b) 2000 intercensal estimates (US CB, 2004d).

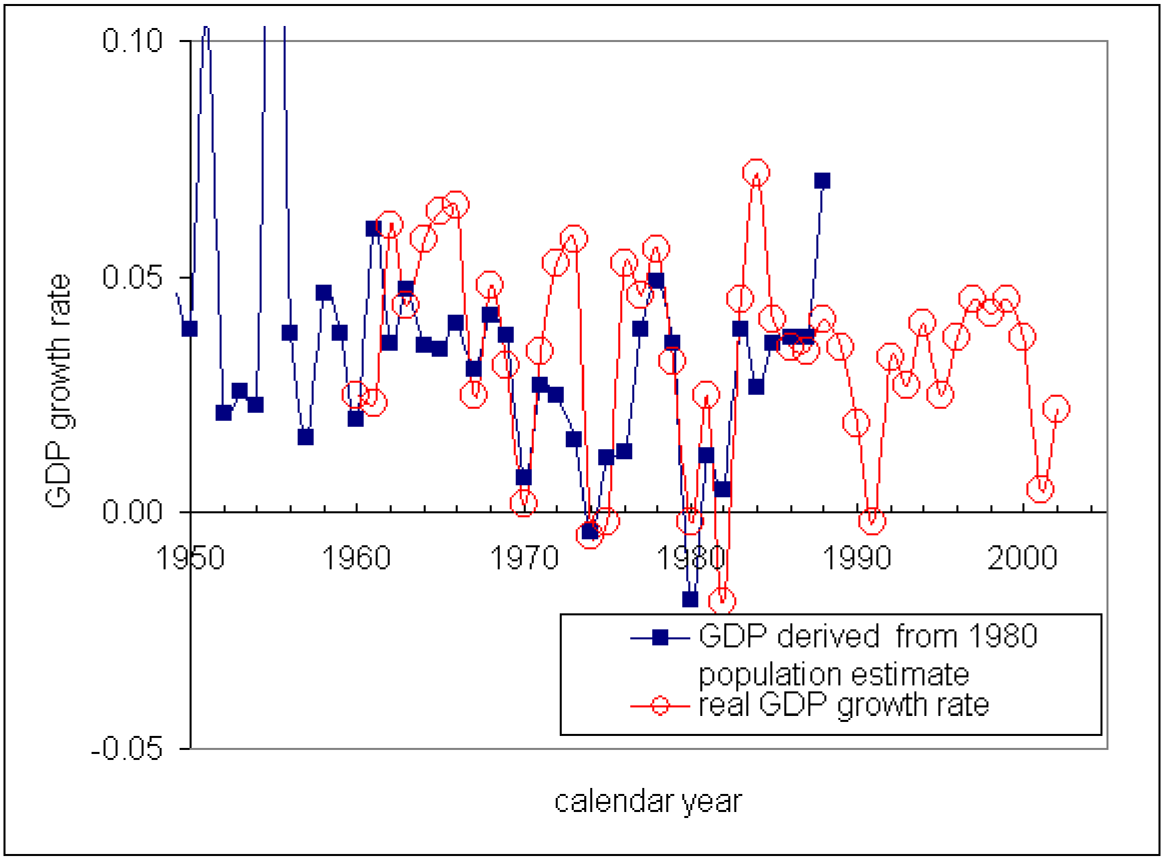


Fig. 2. Real GDP growth rate in the USA for the period from 1960 to 2002.Comparison of the measured values and values predicted by the population estimates of 9-year-olds made as a projection of the 1980 single year of age population estimate into the 9-year-olds number. Notice the predicted and observed curve behaviour around the year of 1980 - oscillations are synchronized. The pattern is smoothed in the subsequent vintages and is not observed in the distribution of the 9-year-olds number.

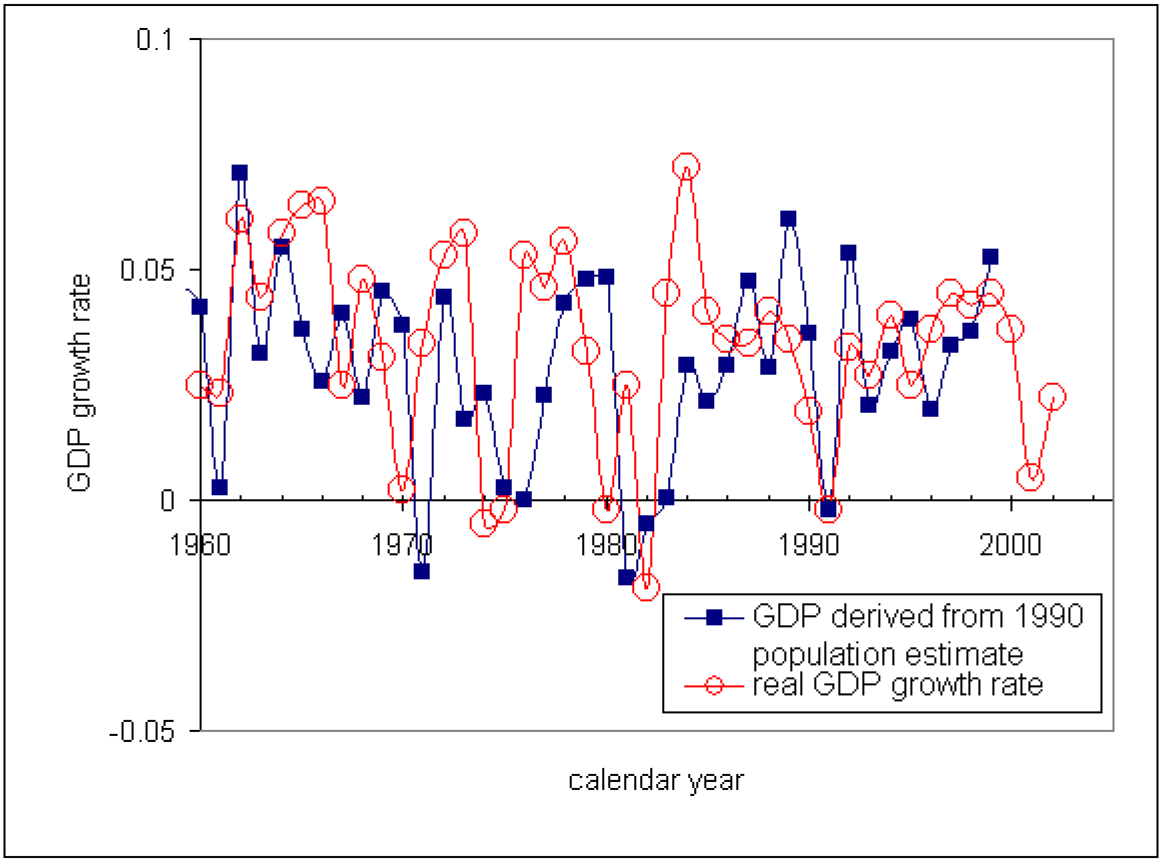


Fig. 3. Real GDP growth rate in the USA for the period from 1960 to 2002.Comparison of the measured values and values predicted by the population estimates of 9-year-olds made as a back projection of the 1990 single year of age population estimate. Notice the predicted and observed curve behaviour around the year of 1990. The oscillations observed near 1980 in Fig. 2 are not observed any more. The oscillations correspond to ages between 17 and 22 in the 1990 estimate.

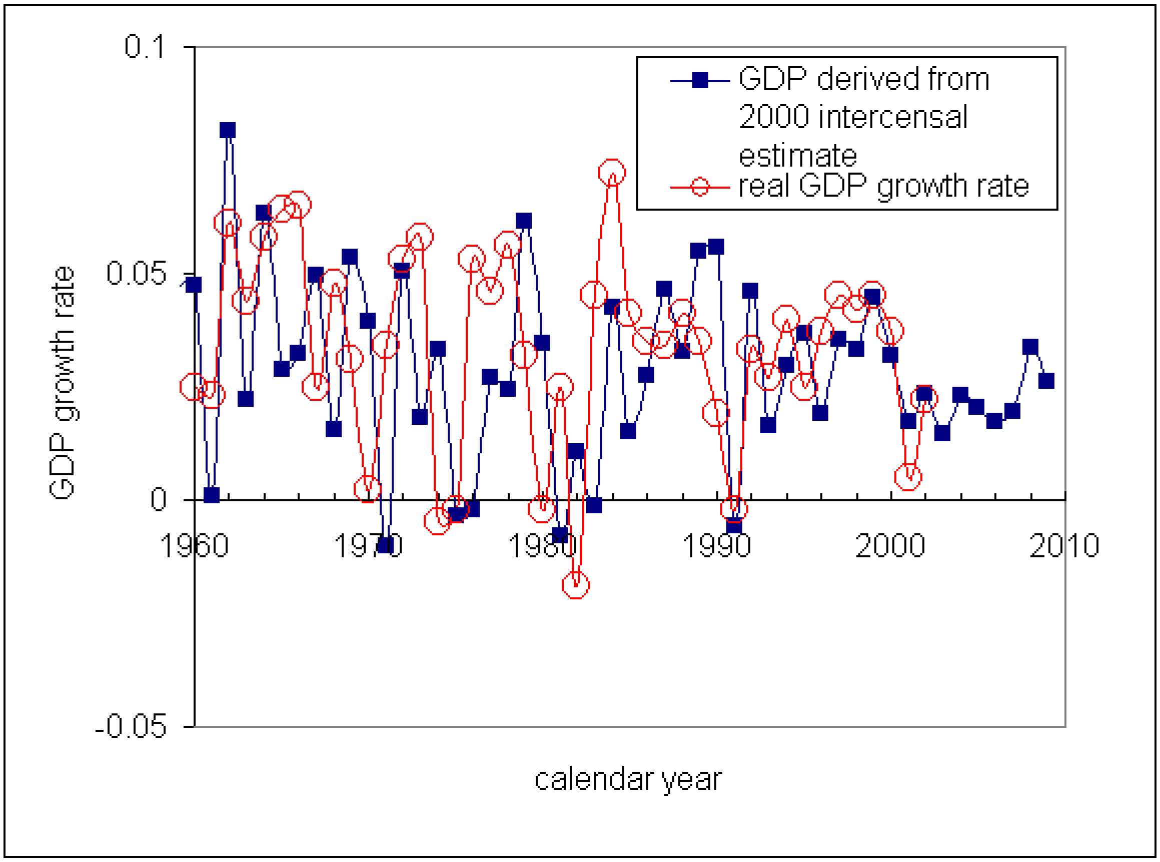


Fig. 4. Real GDP growth rate in the USA for the period from 1960 to 2001.Comparison of the measured values and values predicted by the population estimates of 9-year-olds made as a projection of the 2000 single year of age population ***intercensal*** estimate. Notice the predicted and observed curve behaviour around the years of 1980, 1990, and 2000.

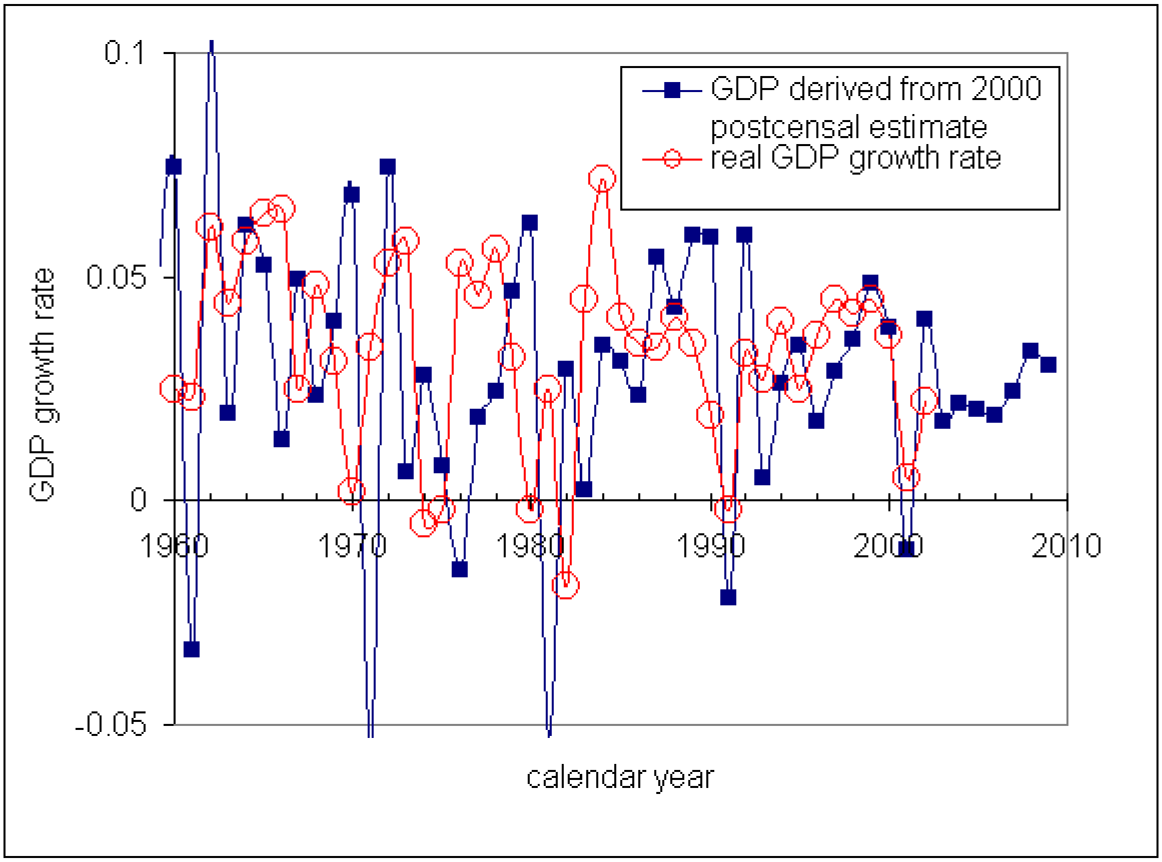


Fig. 5. Real GDP growth rate in the USA for the period from 1960 to 2001.Comparison of the measured values and values predicted by the population estimates of 9-year-olds made as a back projection of the 2000 single year of age population ***postcensal*** estimate. Notice the difference with the intercensal estimates shown in Fig. 4.

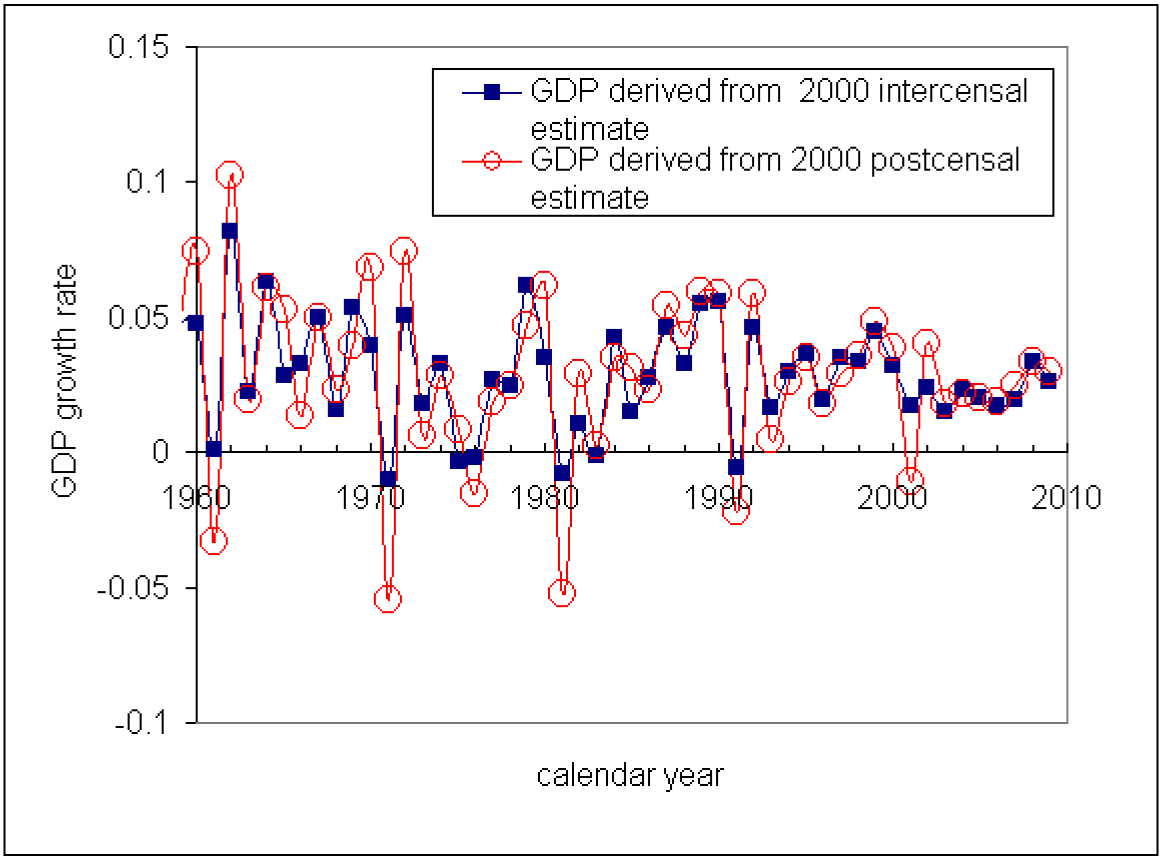


Fig. 6. Predicted GDP growth rate in the USA for the period from 1960 to 2001.Comparison of the values predicted by the population estimates of 9-year-olds made as a back projection of the 2000 single year of age population ***postcensal*** and ***intercensal*** estimates. The latter is a severely smoothed version of the former.

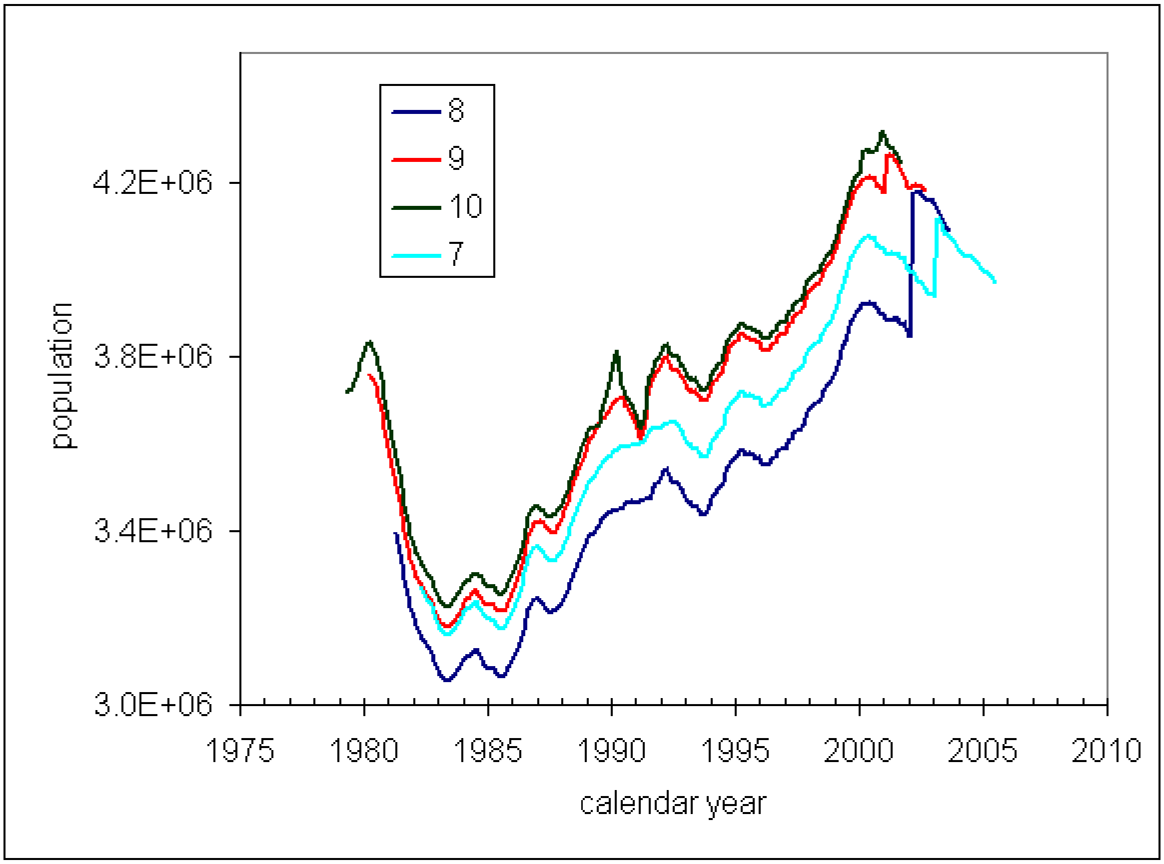


Fig. 7. Comparison of the quarterly (before 1990) and monthly (after 1990) ***postcensal*** estimates of various single year of age population: 7, 8, 9, and 10 years. The distributions are shifted in time relative to the 9-year-olds curve in order to trace the same cohort. Notice the difference between 8-year-olds and 9-year-olds reaching 300,000. The number of 7-year-olds is consistently above the number of 8-year-olds violating the general rule of increasing cohort population by single year of age.

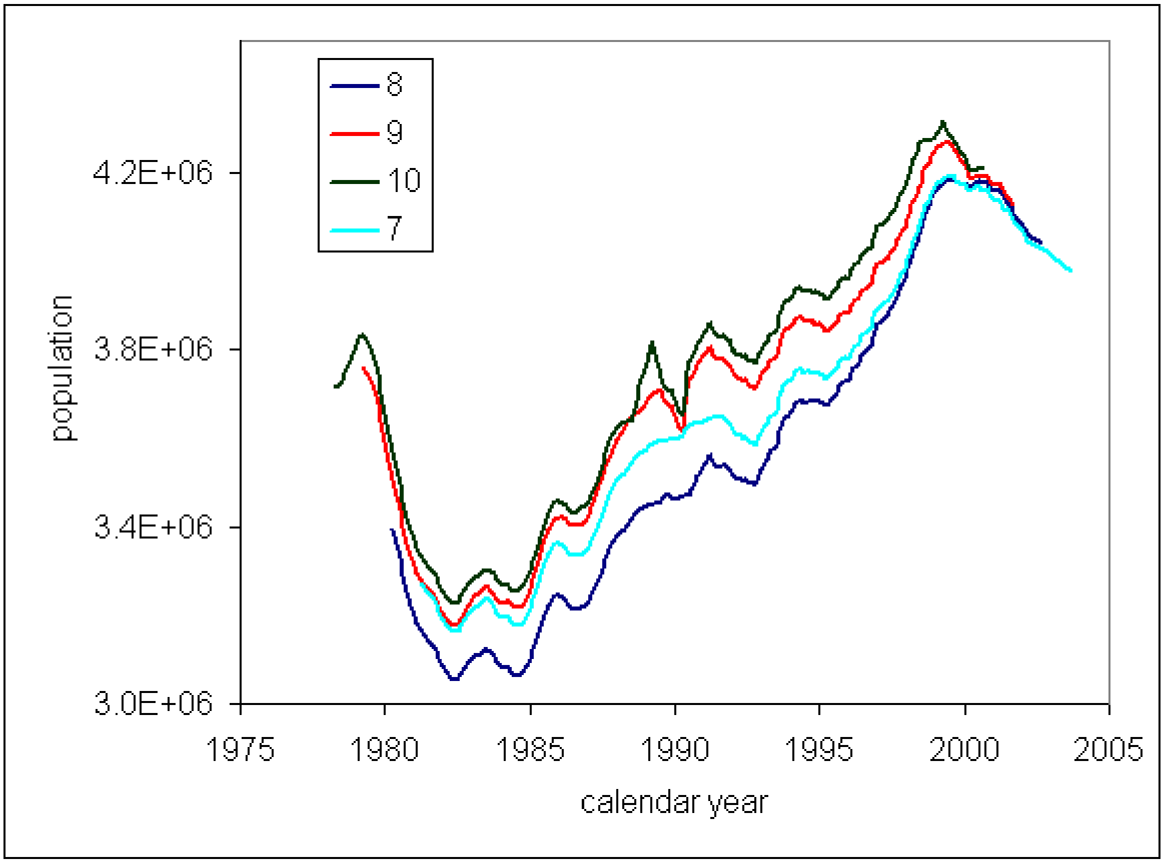


Fig. 8. Same as in Fig. 6 for the ***intercensal*** (1980-2000) estimates. Notice the convergence of the curves for the 7-year-olds and 8-year-olds in comparison with the postcensal estimates. This correction gives an approximate estimate of the uncertainty of the 9-year-olds number. One can not distinguish between curves inside this uncertainty bounds.

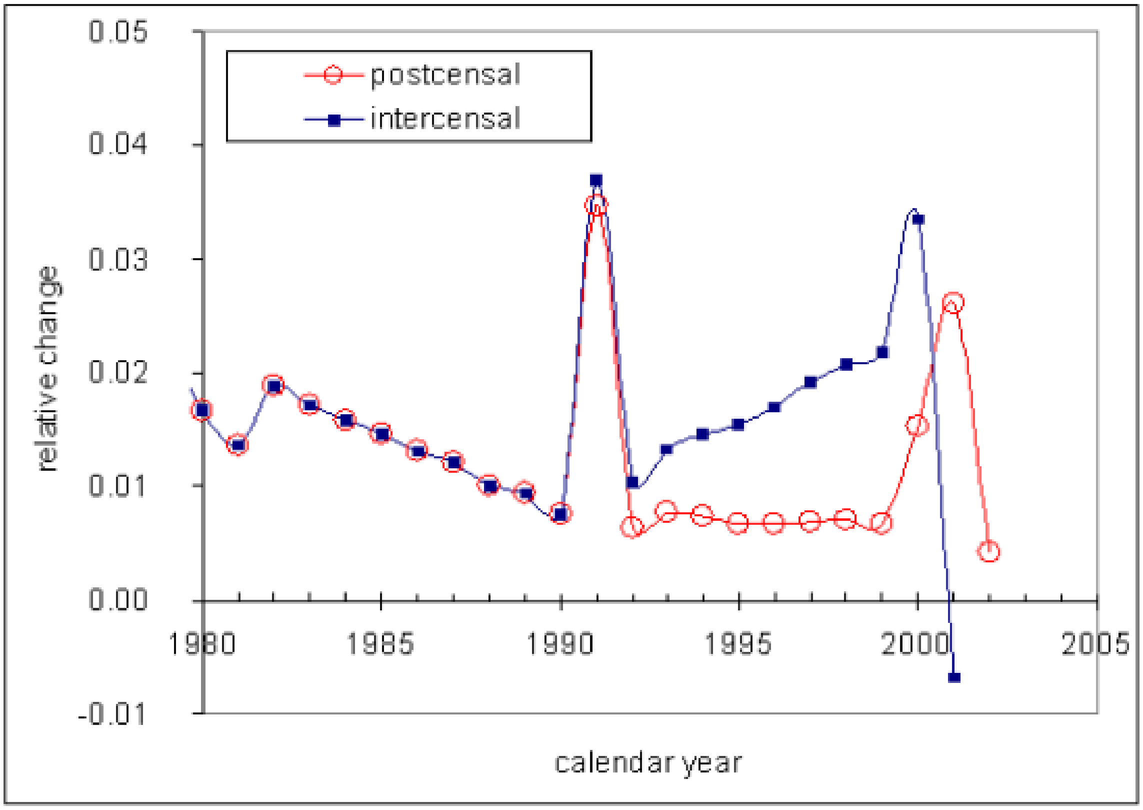


Fig. 9. Change of 9-year-old population during one year, i.e. the difference between 9-year-olds and 10-year-olds in the next year. Notice severe population changes near the census years and linear growth of the difference between censuses.

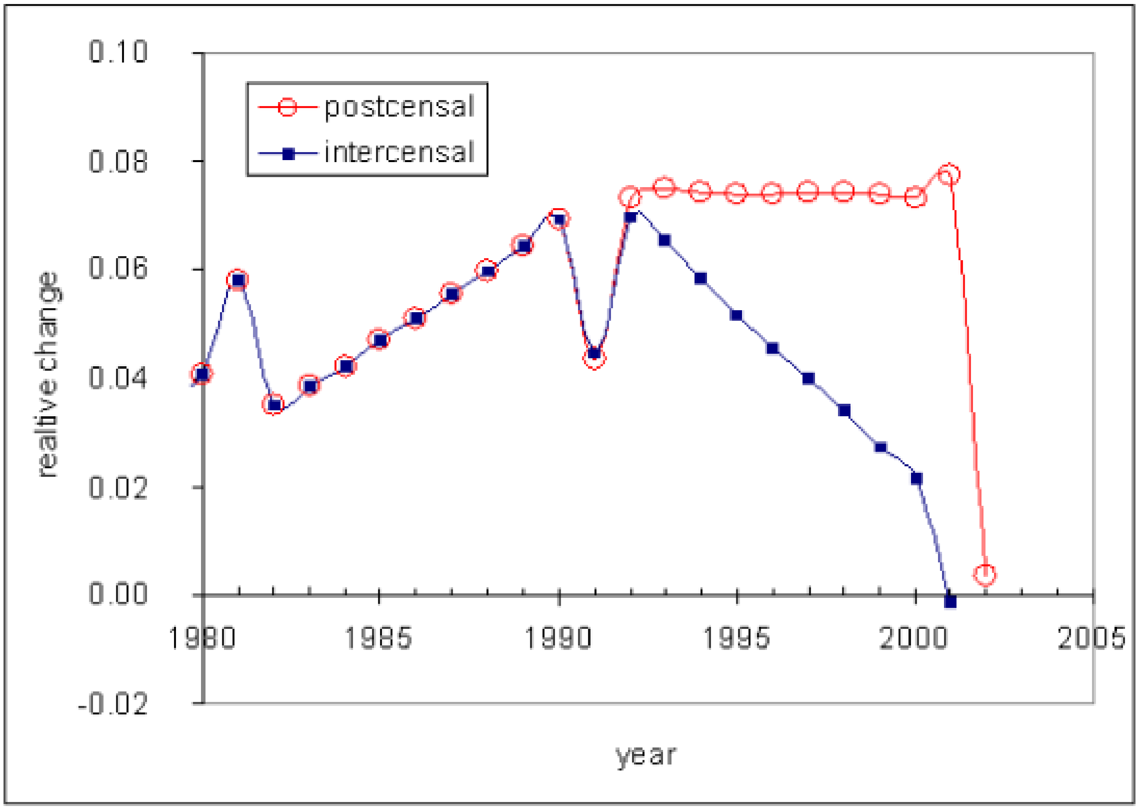


Fig. 10. Same as in Fig. 9 for the 8-year-olds. Notice two corrections in 1981 and 1990. One can suppose the uncertainty of the 9-year-olds number of several percent.

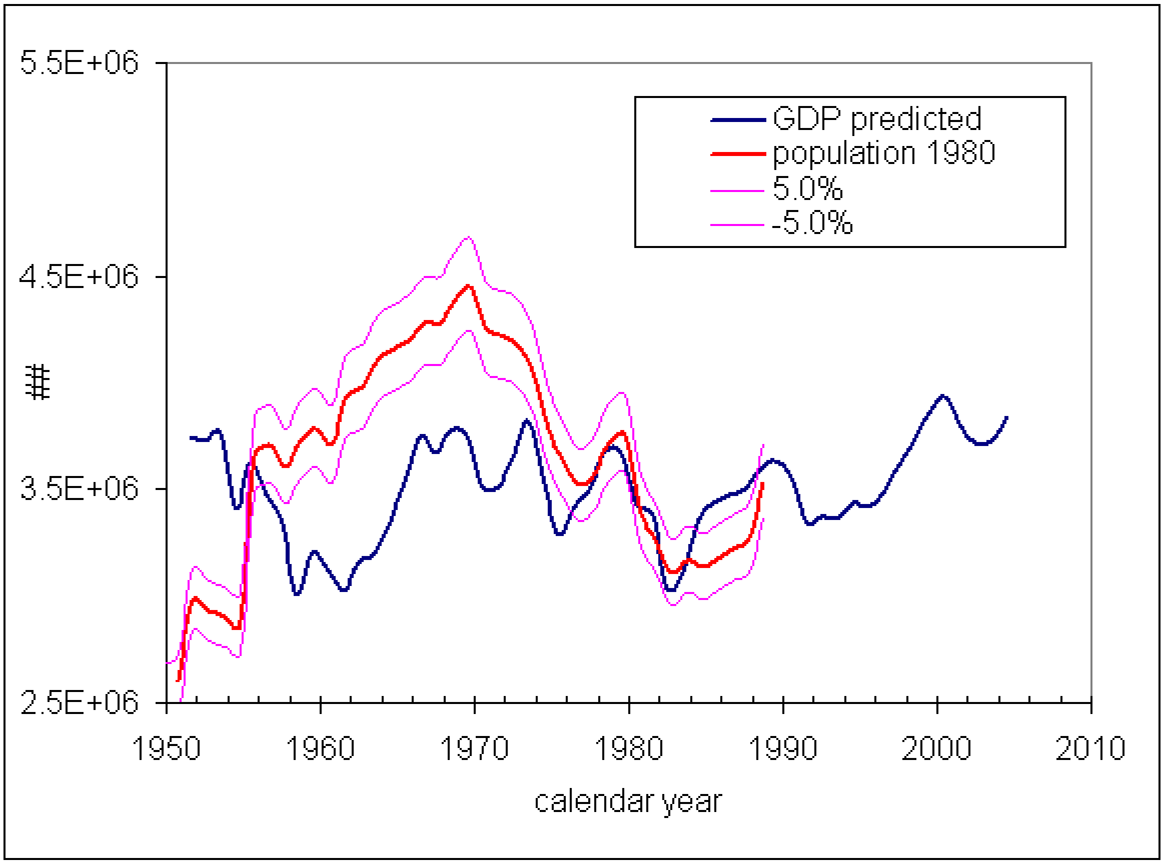


Fig. 11. Comparison of the 9-year-old population as obtained by a back projection from the 1980 population estimate and the number of 9-year-olds obtained from the real GDP observations according to equation (3). Notice the coincidence of the curves near 1980. The initial value of the 9-year-olds in 1951 is 3750000. The actual number of 9-year-olds is obviously overestimated by the projection for the period before 1975 due to positive demographic dynamics in the USA.

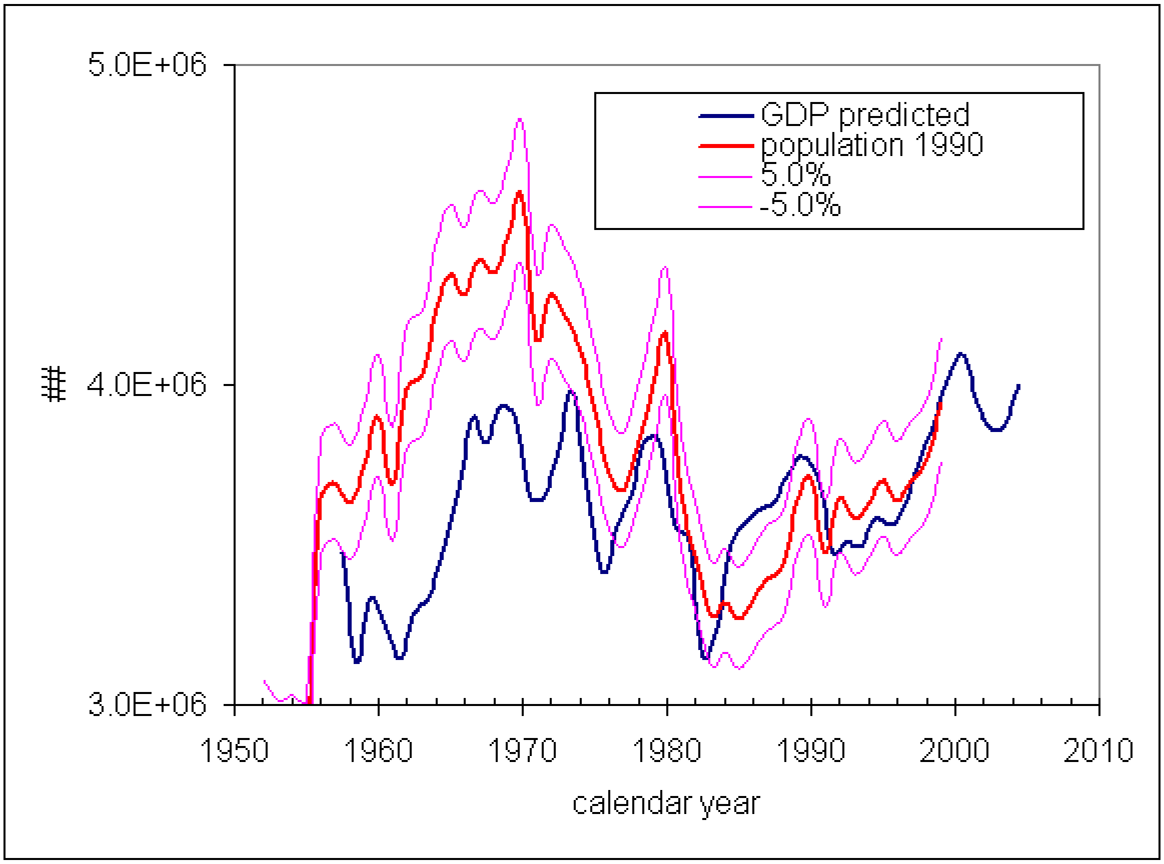


Fig. 12. Comparison of the 9-year-old population as obtained by a back projection from the 1990 population estimate and the number of 9-year-olds obtained from the real GDP observations according to equation (3). Notice the estimated curve behaviour near 1990 - the observed oscillation is considerably smoothed in following estimates and in the estimates of 9-year-olds. The initial value of the 9-year-olds in 1951 is 3900000. The actual number of 9-year-olds is overestimated by the projection for the period before 1985 due to positive demographic dynamics in the USA.

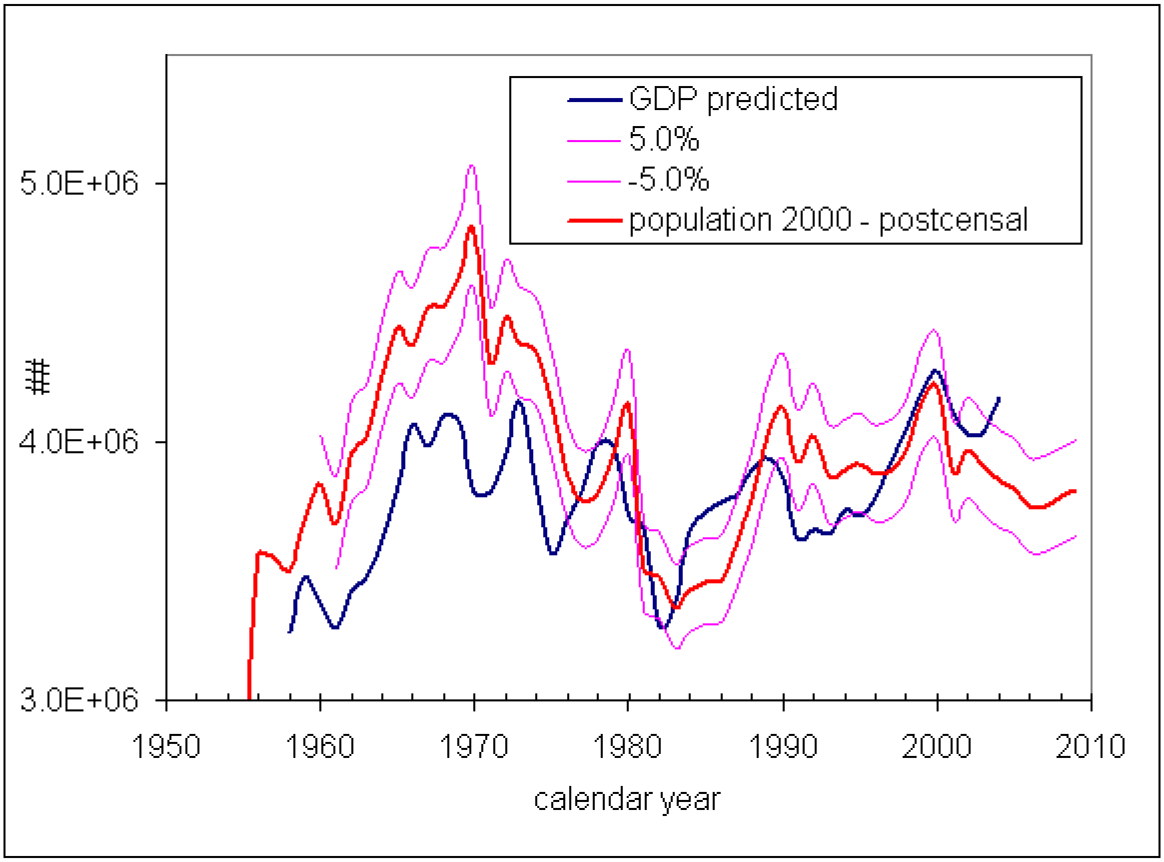


Fig. 13. Comparison of the 9-year-old population as obtained by a back projection from the 2000 postcensal population estimate and the number of 9-year-olds obtained from the real GDP observations according to equation (3). Notice the coincidence of the curves near 2000. The initial value of the 9-year-olds in 1951 is 3800000. The actual number of 9-year-olds is overestimated by the projection for the period before 1975 but in excellent agreement between 1975 and 2001.

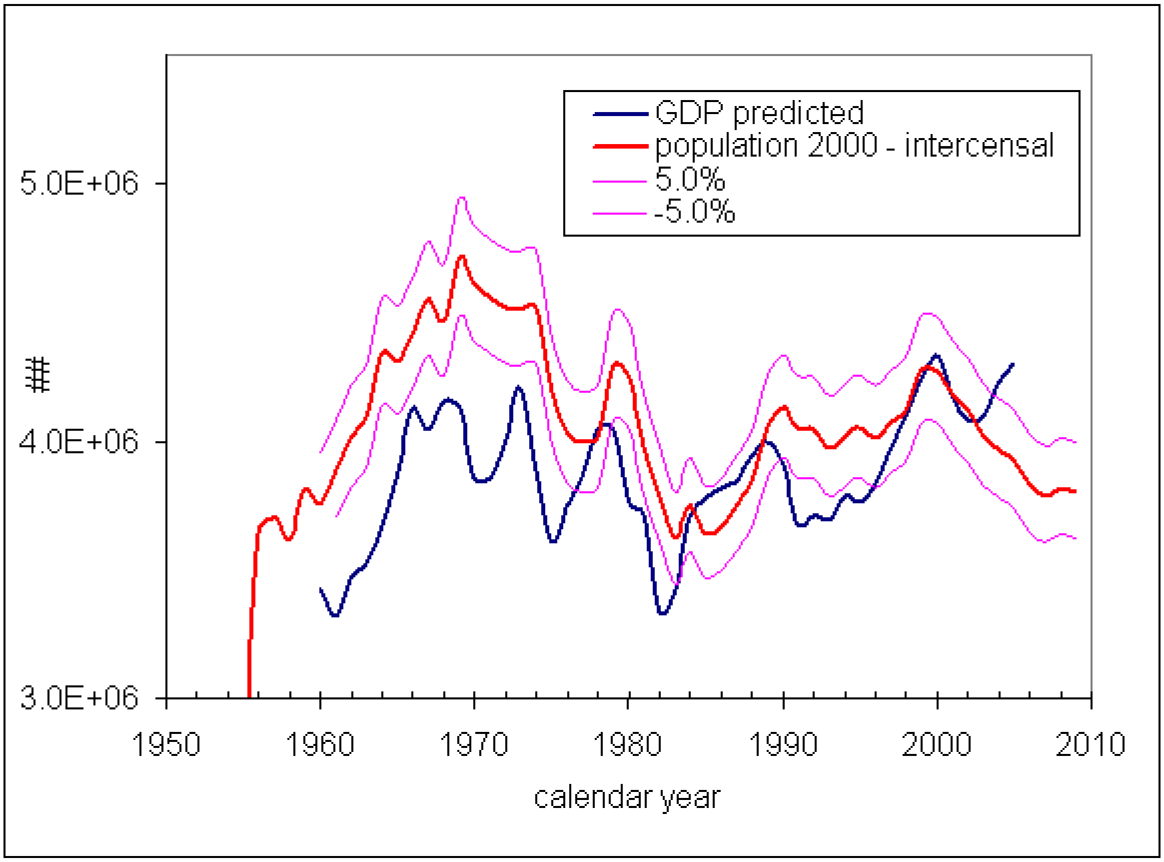


Fig. 14. Comparison of the 9-year-old population as obtained by a back projection from the 2000 intercensal population estimate and the number of 9-year-olds obtained from the real GDP observations according to equation (3). Notice the coincidence of the curves near 2000. The initial value of the 9-year-olds in 1951 is 3800000. The actual number of 9-year-olds is overestimated by the projection for the period before 1975 but in excellent agreement between 1975 and 2001.

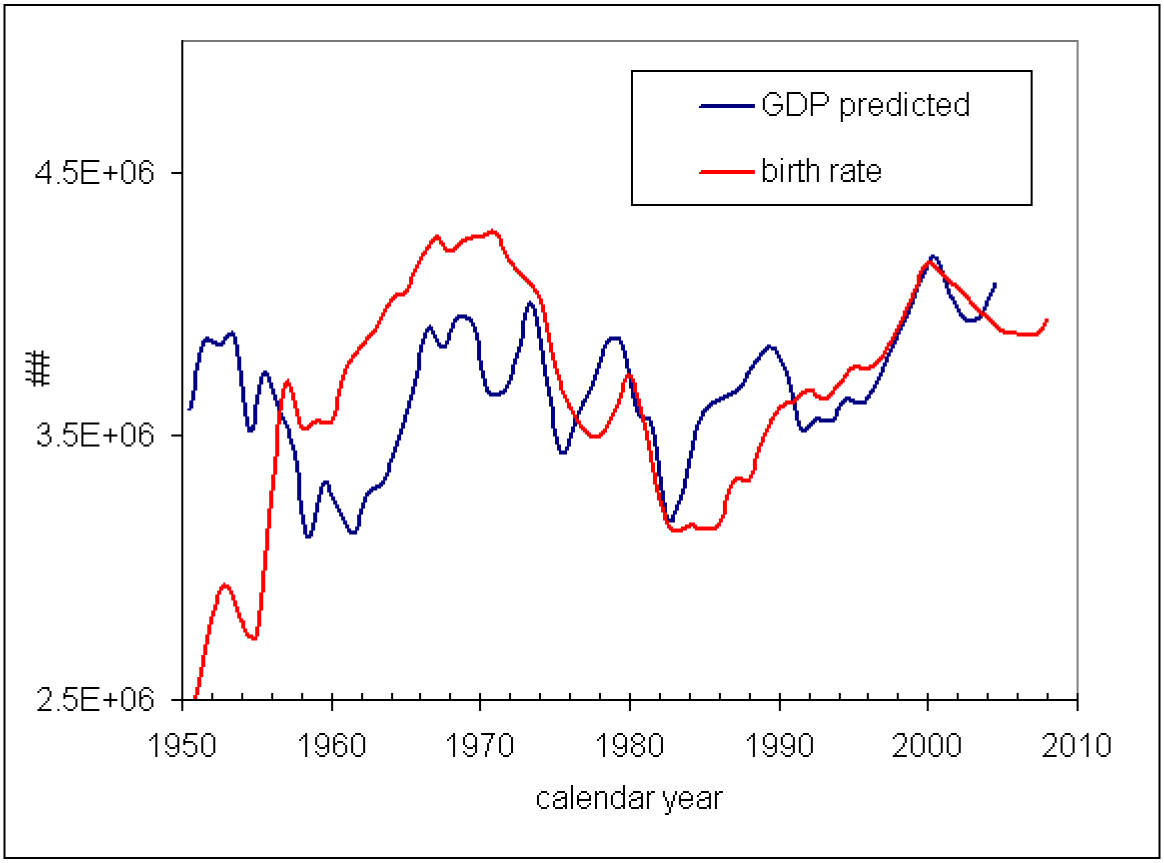
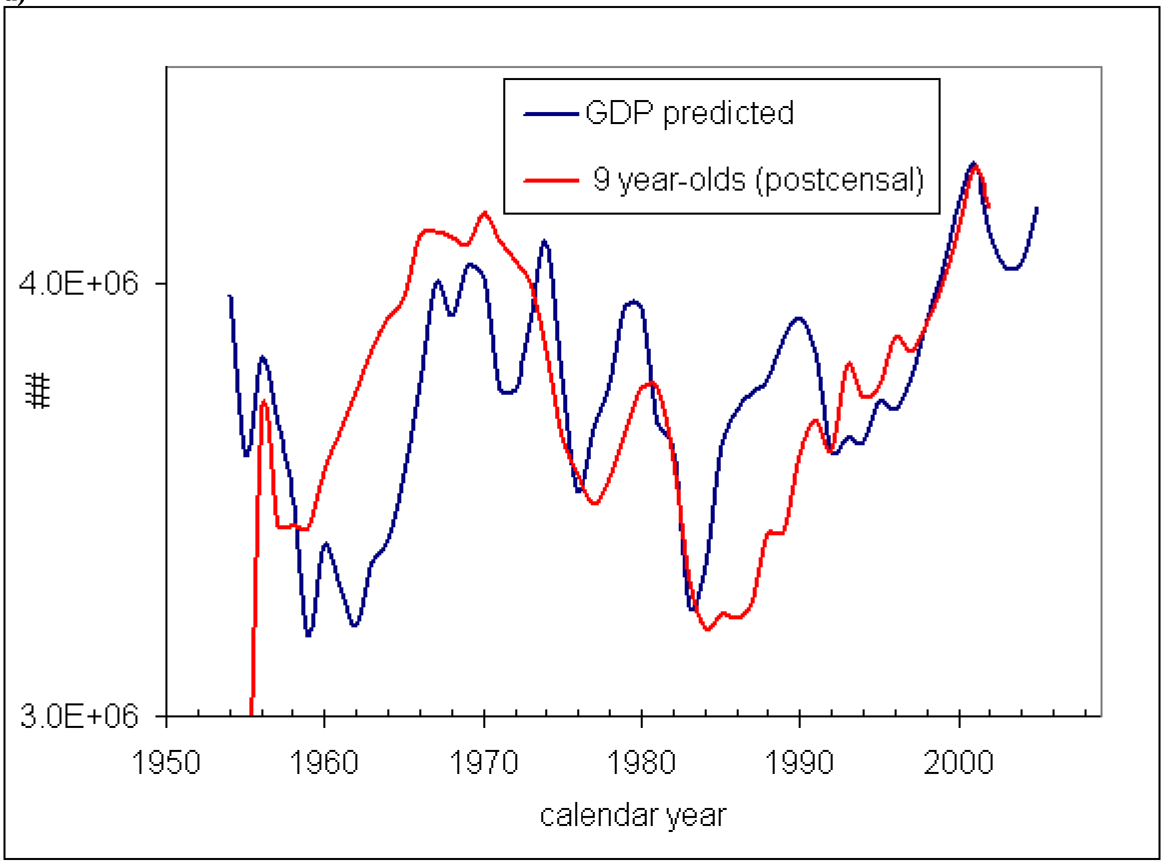


Fig. 15. Comparison of the 9-year-old population as obtained by a back projection from the birth rate (shifted by 9 years ahead) and the number of 9-year-olds obtained from the real GDP observations according to equation (3). Notice a good agreement between the curves. The initial value of the 9-year-olds in 1951 is 3860000. The actual number of 9-year-olds is slightly overestimated by the projection for the period before 1970 but in excellent agreement between 1975 and 2001. No oscillation of the 9-year-olds near 1990 as observed in 1990 population estimate.

a)



GDP predicted
9 year-olds (postcensal)
4.0E+06
#
3.0E+06
1950
1960
1970
1980
1990
2000
calendar year

b)

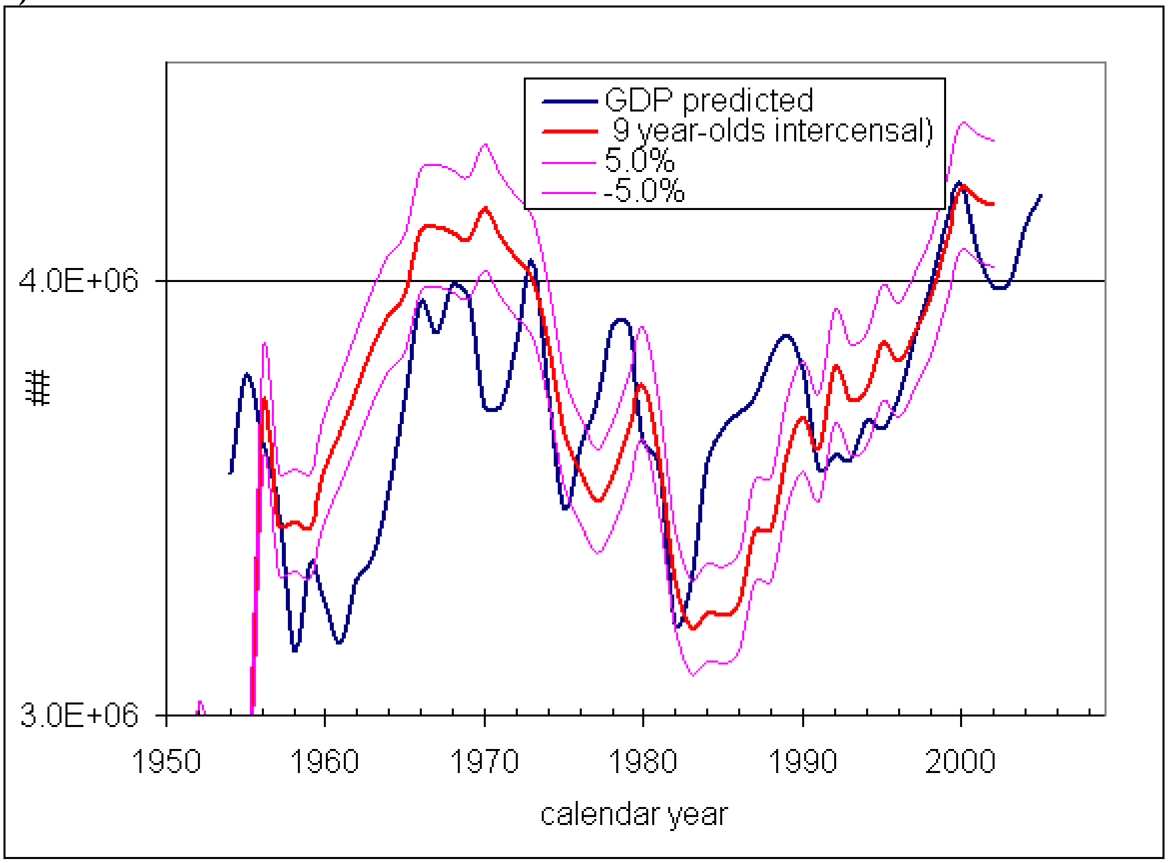


Fig. 16. Comparison of the 9-year-old population - a)postcensal estimate and b) intercensal estimate - and the number of 9-year-olds obtained from the real GDP observations according to equation (3). Notice a good agreement between the curves. The actual number of 9-year-olds is slightly overestimated by the projection for the period before 1970 but is in a good agreement between 1975 and 2001.

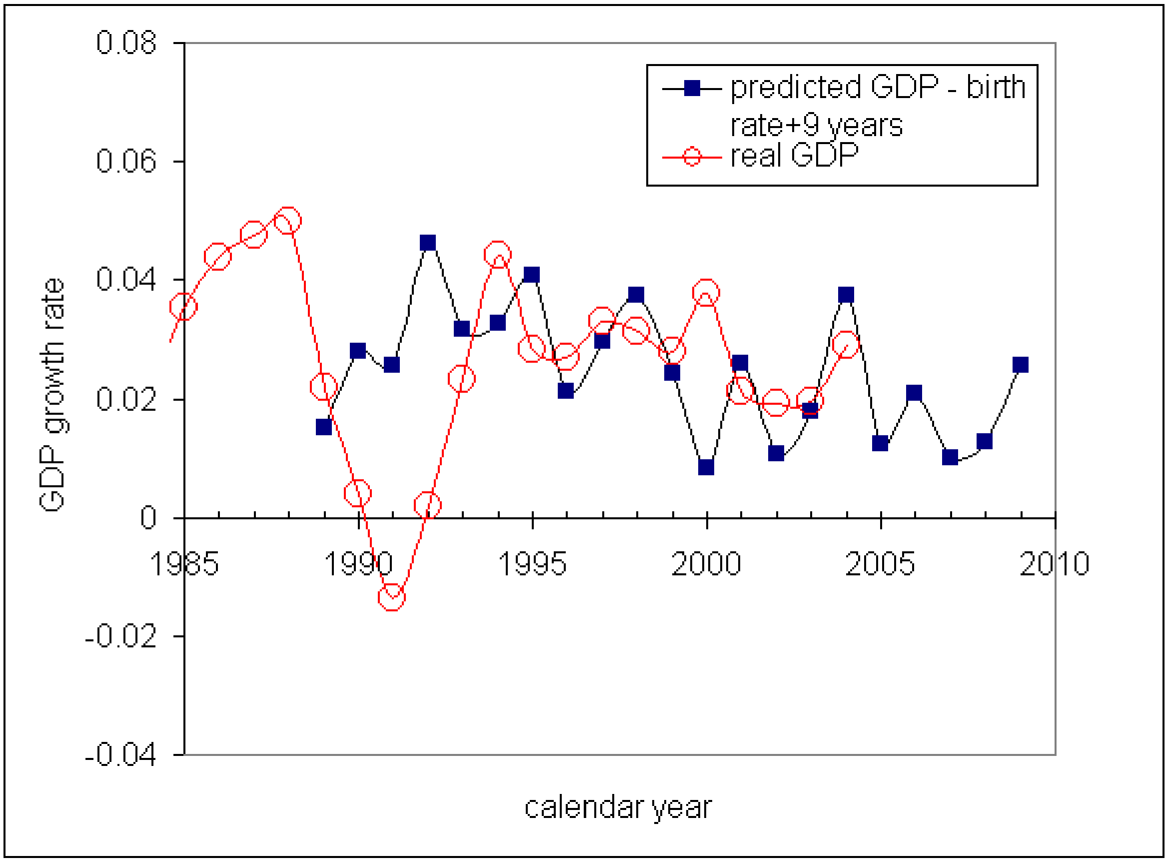


Fig. 17. Real GDP growth rate in the UK for the period between 1985 and 2004. Comparison of the measured values and values predicted by the single year of age population estimates (birth rate + 9 years of life):

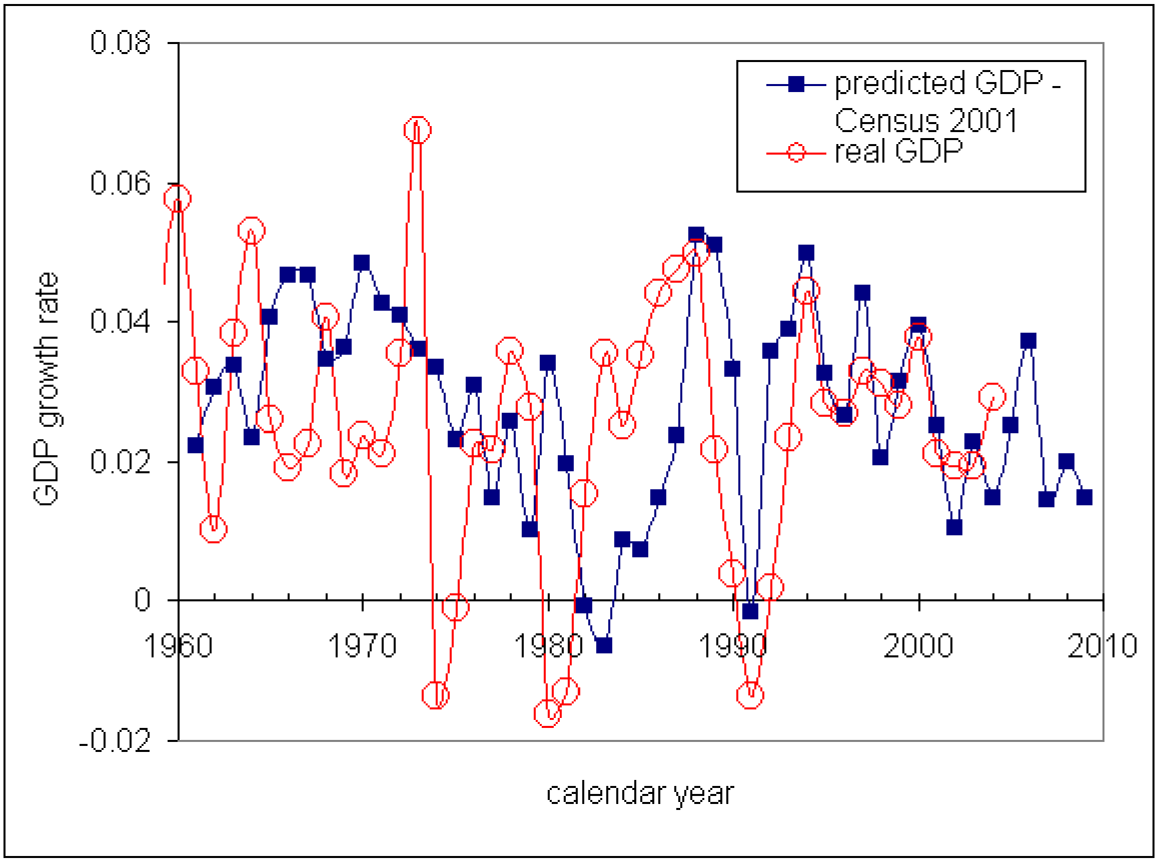


Fig. 18. Real GDP growth rate in the UK for the period between 1960 and 2004. Comparison of the measured values and values predicted by the single year of age population estimates (back projection of the 2001 census).

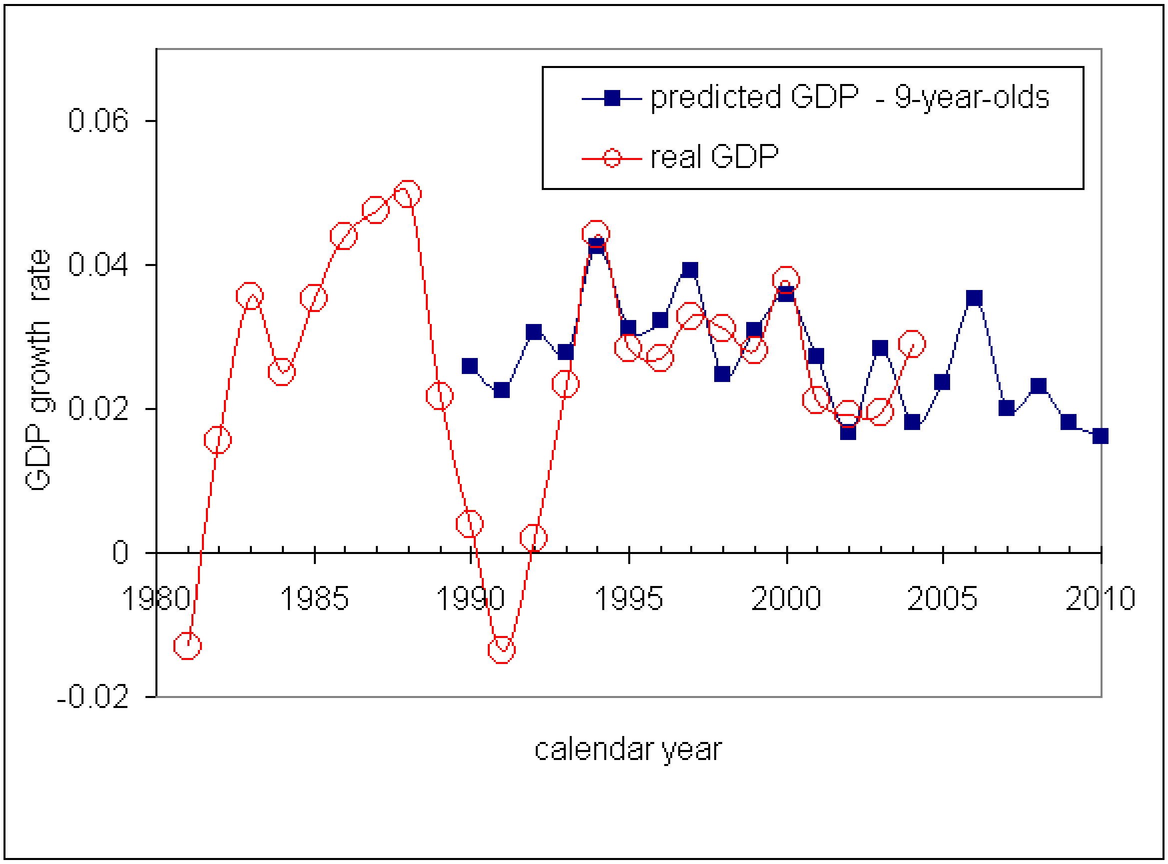


Fig. 19. Real GDP growth rate in the UK for the period between 1990 and 2004. Comparison of the measured values and values predicted by the 9-year-olds estimates (.

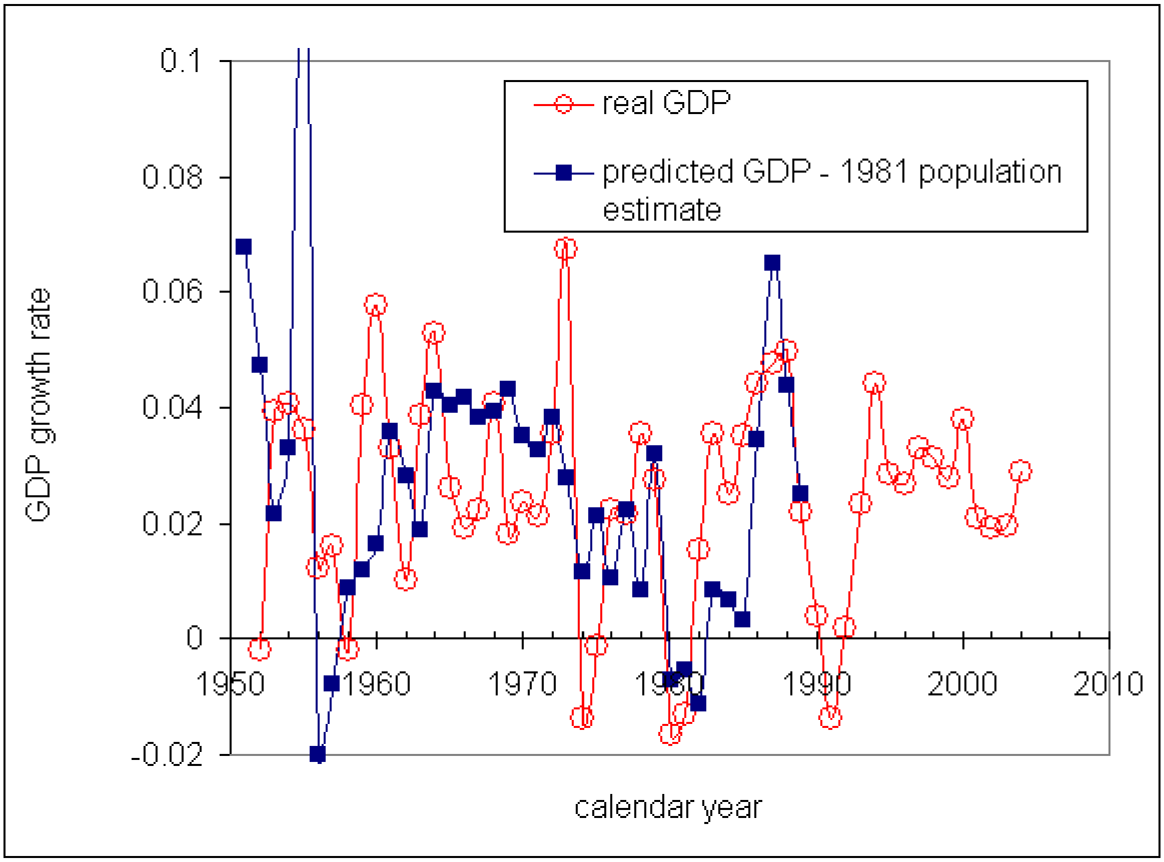


Fig. 20. Real GDP growth rate in the UK for the period between 1950 and 2004. Comparison of the measured values and values predicted by the single year of age population estimates (back projection of the 1981 population estimate). Notice the years around 1980.

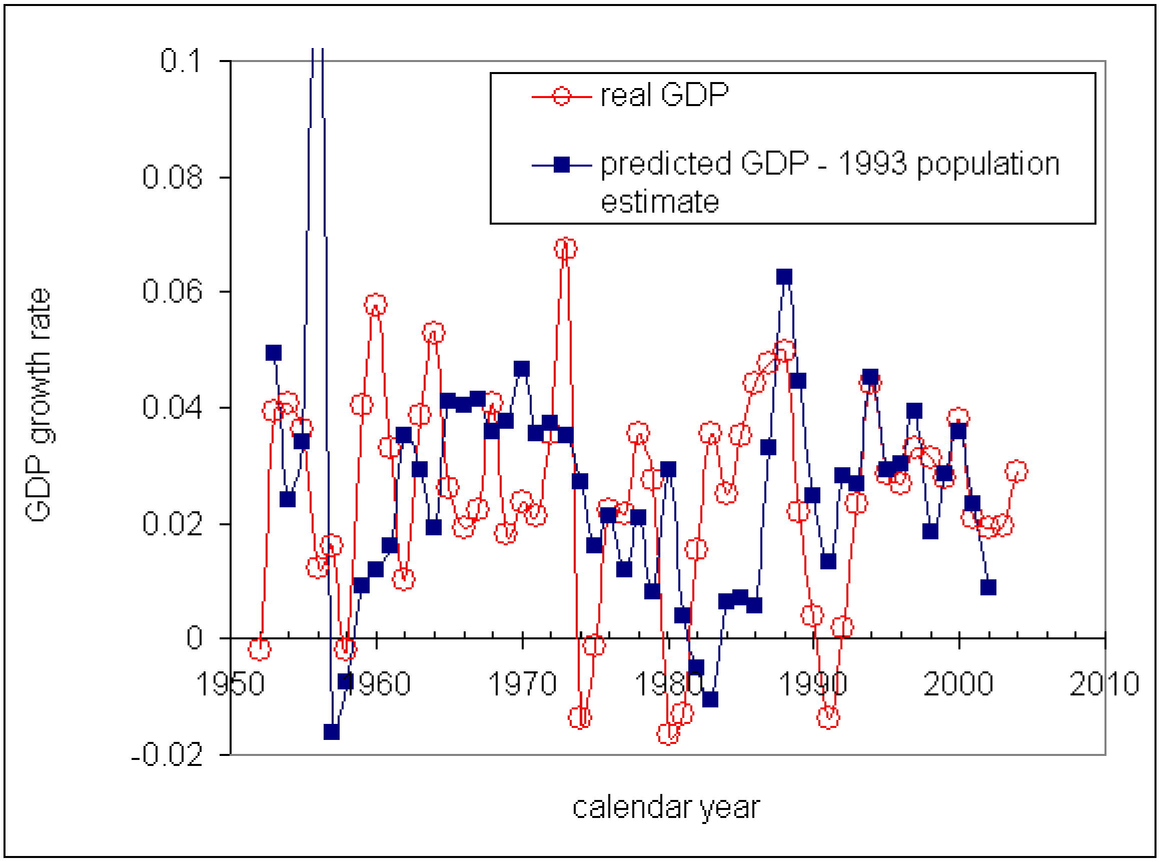
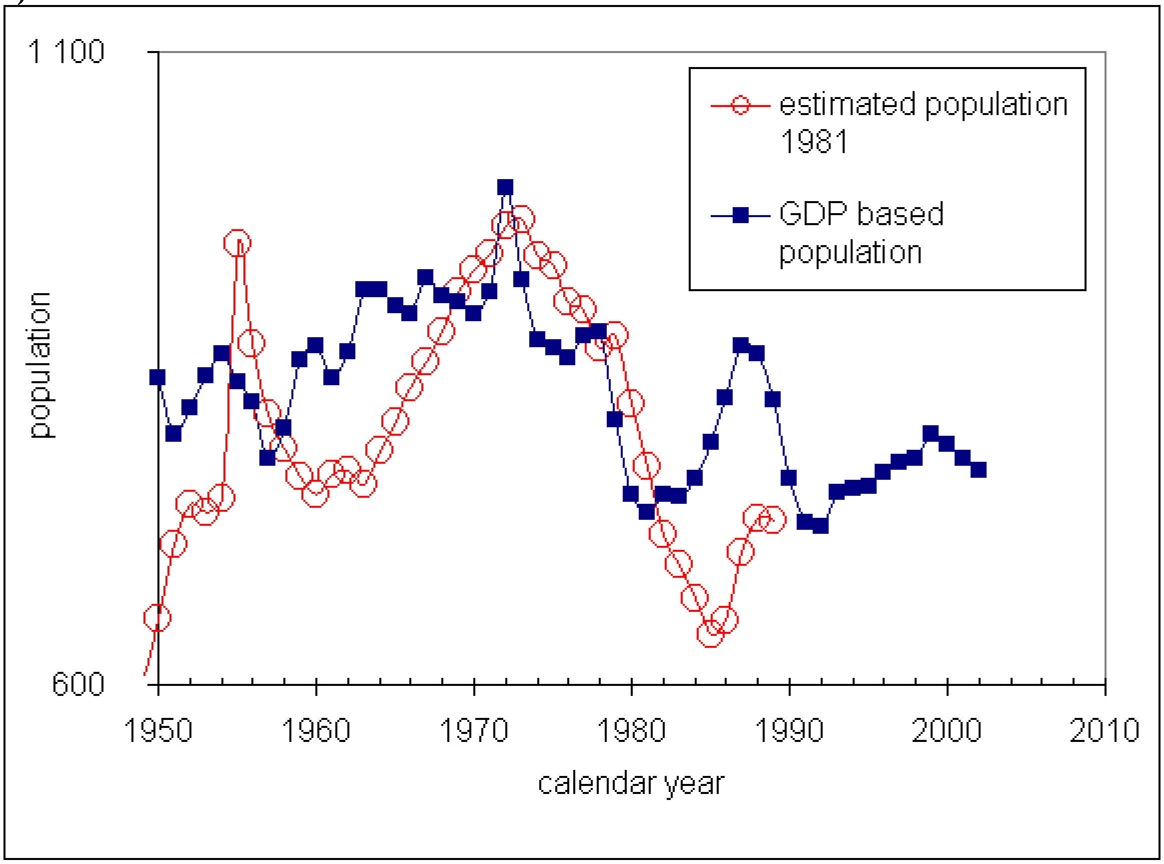
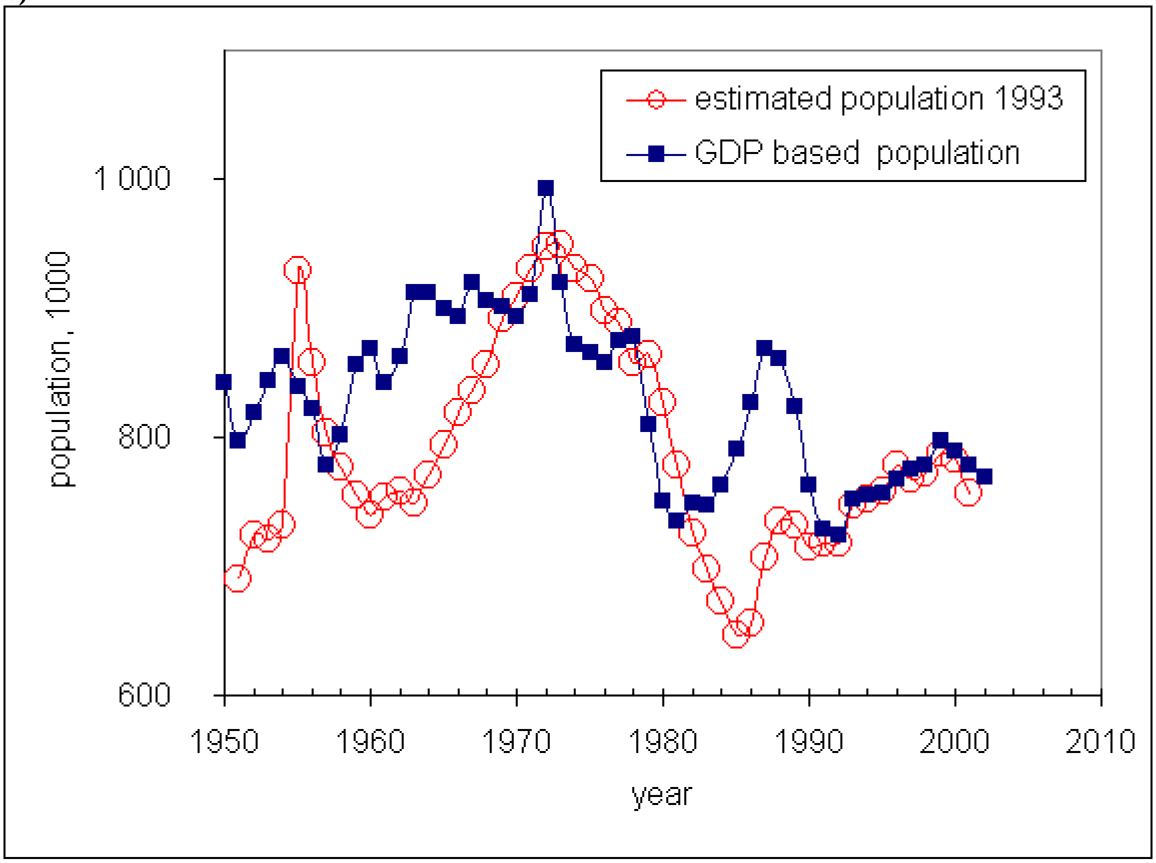


Fig. 21. Real GDP growth rate in the UK for the period between 1950 and 2004. Comparison of the measured values and values predicted by the single year of age population estimates (back projection of the 1993 population estimate). Notice the years around 1990.

a)


1 100
600
population
1950
1960
1970
1980
1990
2000
2010
calendar year
estimated population 1981
GDP based population

b)



estimated population 1993
GDP based population
population, 1000
1 000
800
600
1950
1960
1970
1980
1990
2000
2010
year

c)

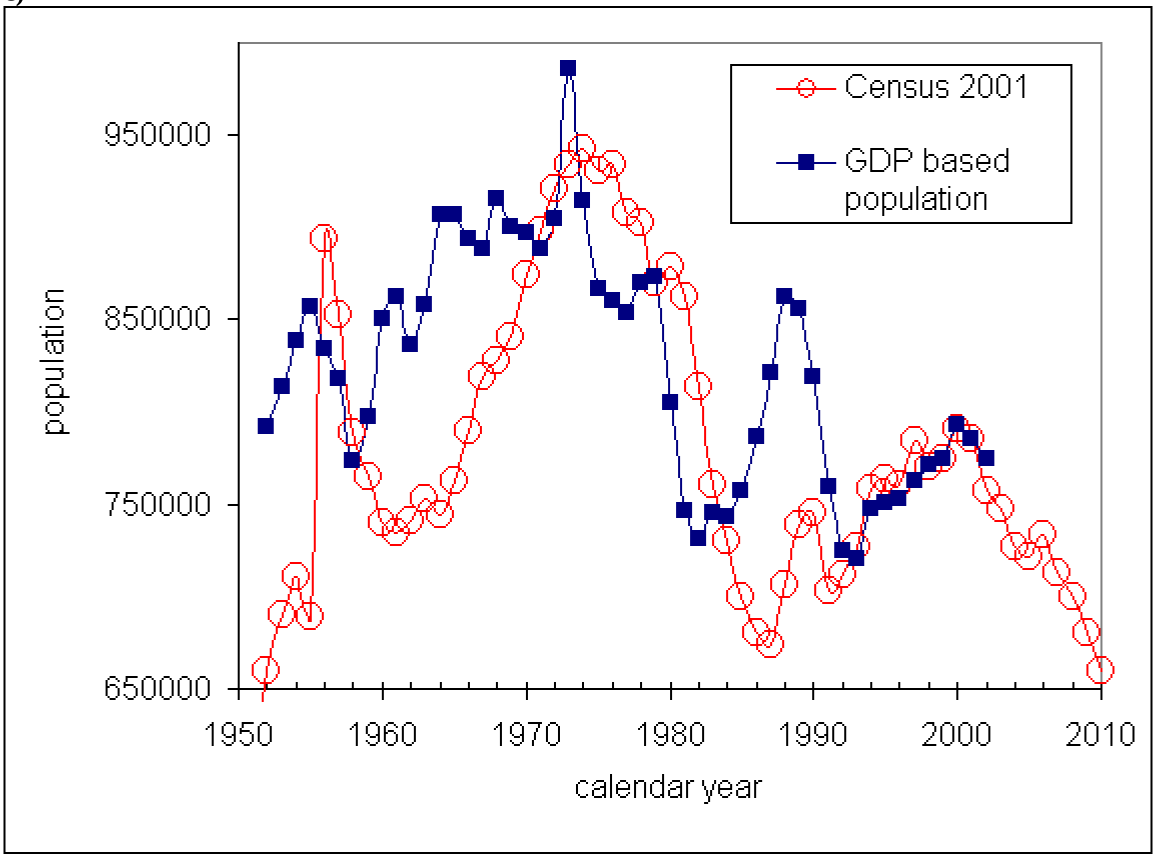


Fig. 22. Comparison of the 9-year-old population in the UK as obtained by a back projection from the 1981 population estimate -**a)**; 1993 population estimate - **b)** and 2001 census -**c)** and the number of 9-year-olds obtained from the real GDP observations according to equation (3).

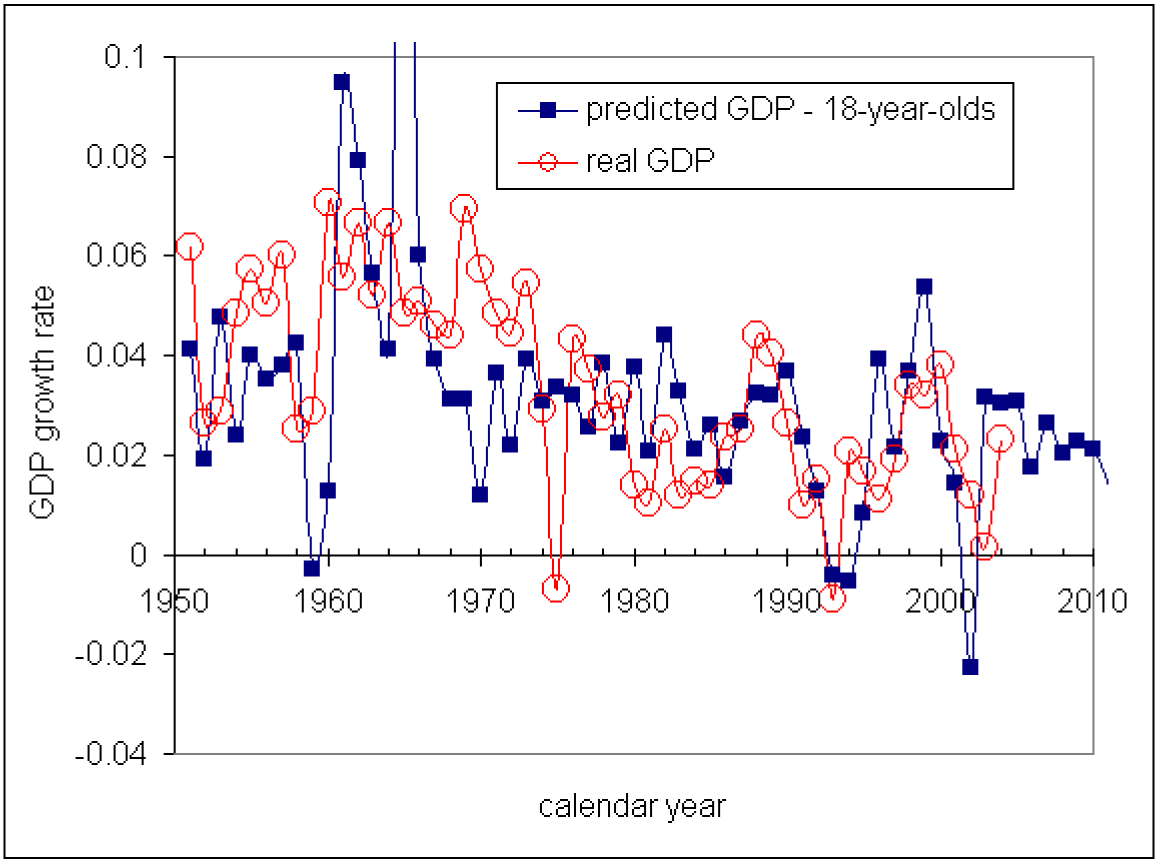


Fig. 23. Real GDP growth rate in France for the period between 1950 and 2004. Comparison of the measured values and values predicted by the single year of age (18-year-olds) population estimates

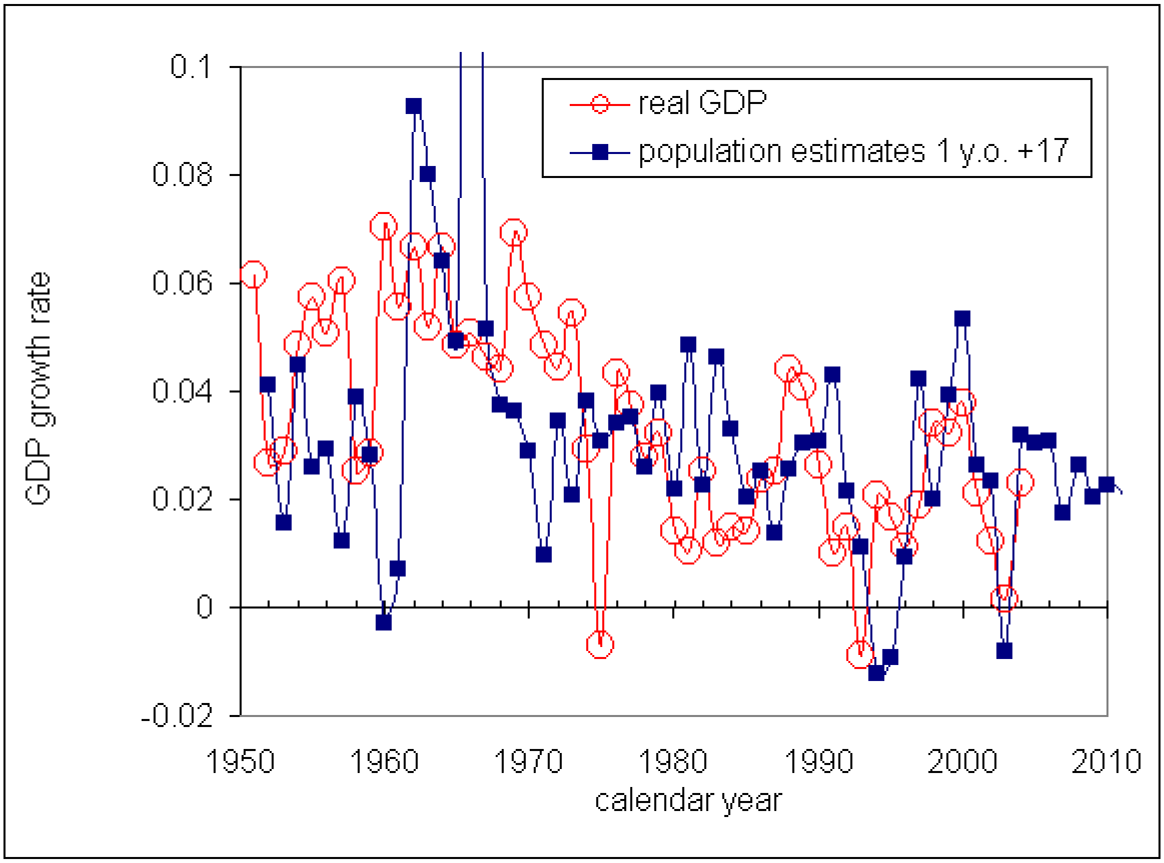


Fig. 24. Real GDP growth rate in France for the period between 1950 and 2004. Comparison of the measured values and values predicted by the single year of age (1-year-olds shifted ahead by 17 years) population estimates

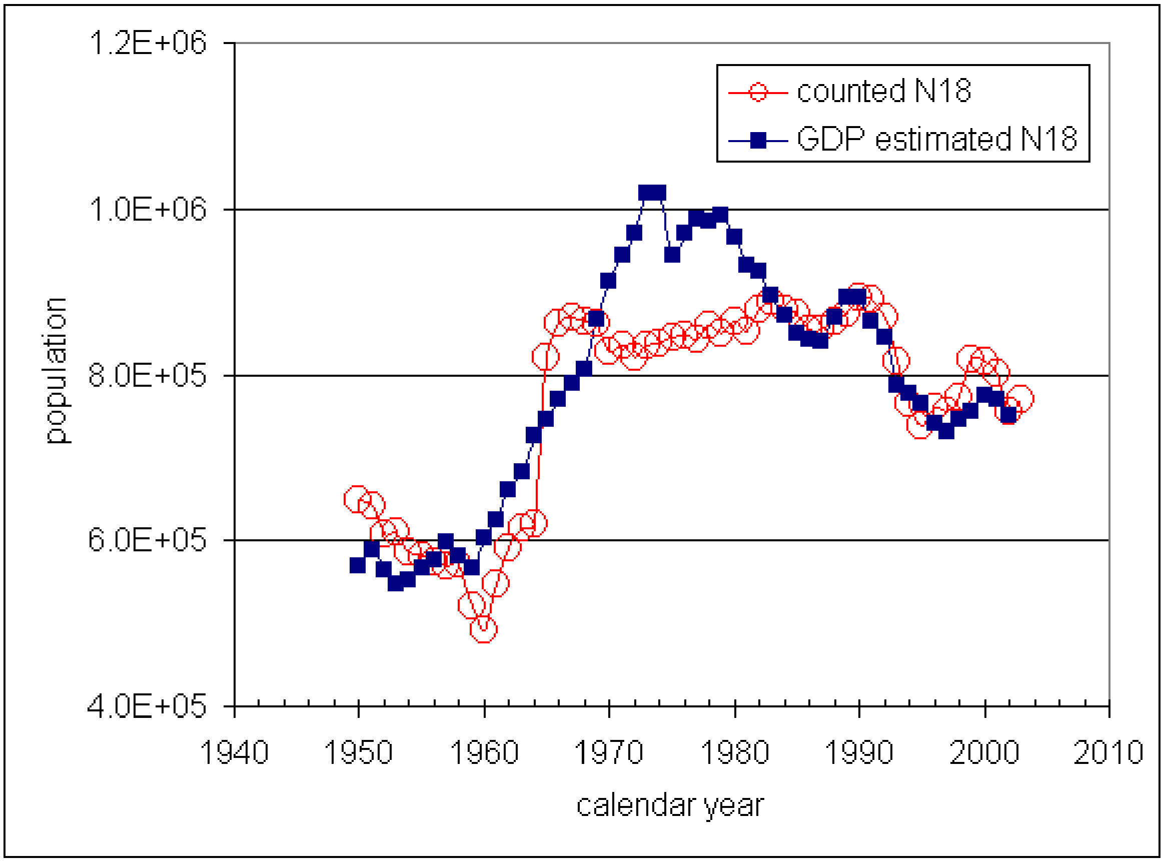
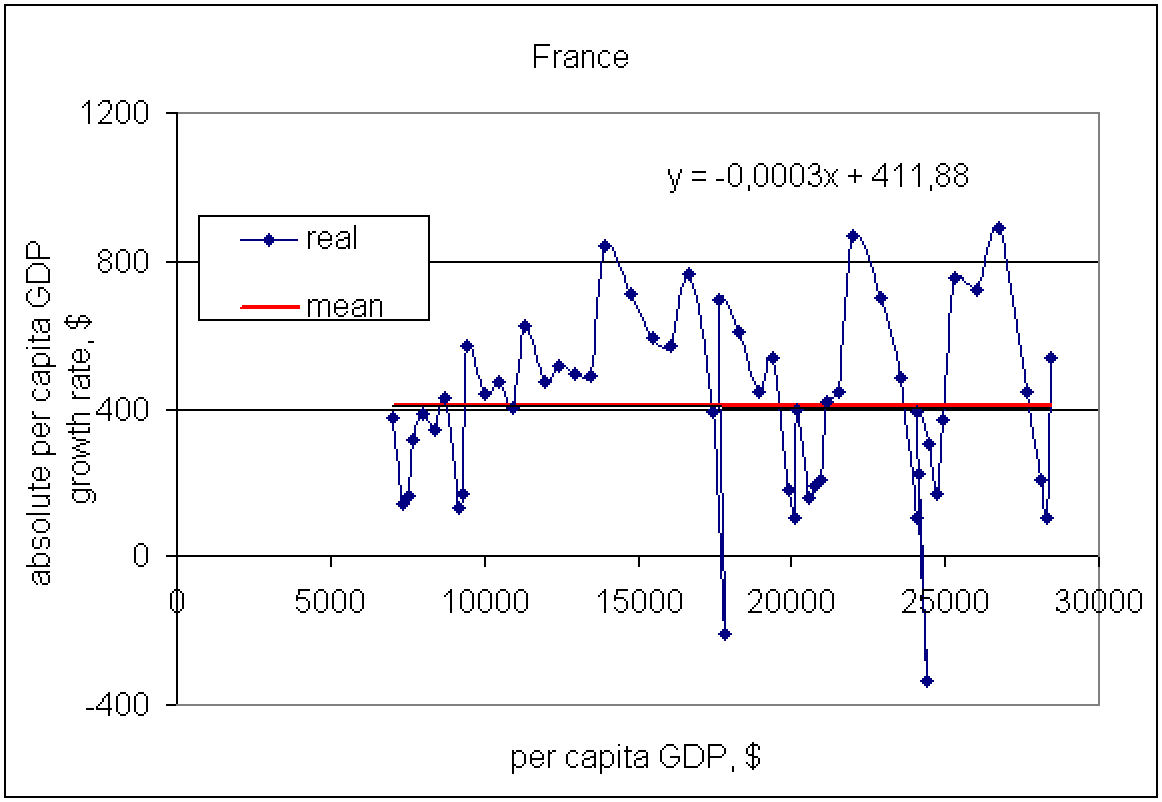
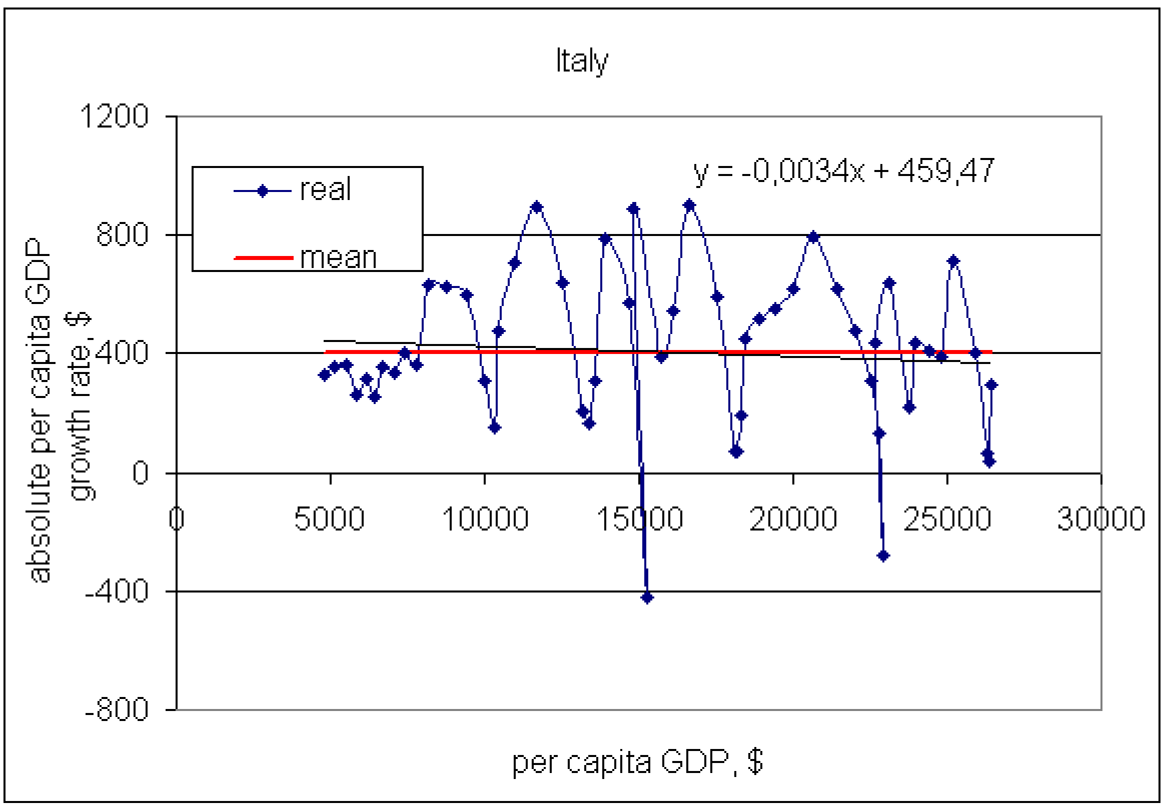
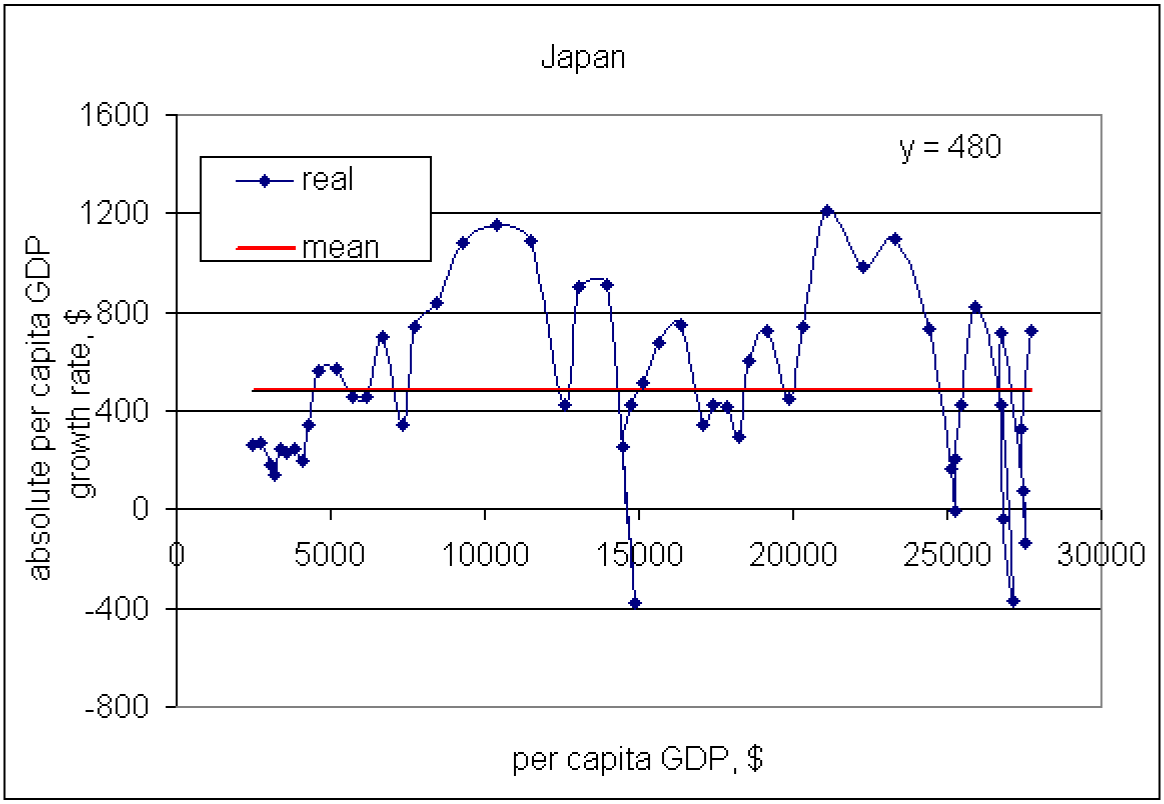
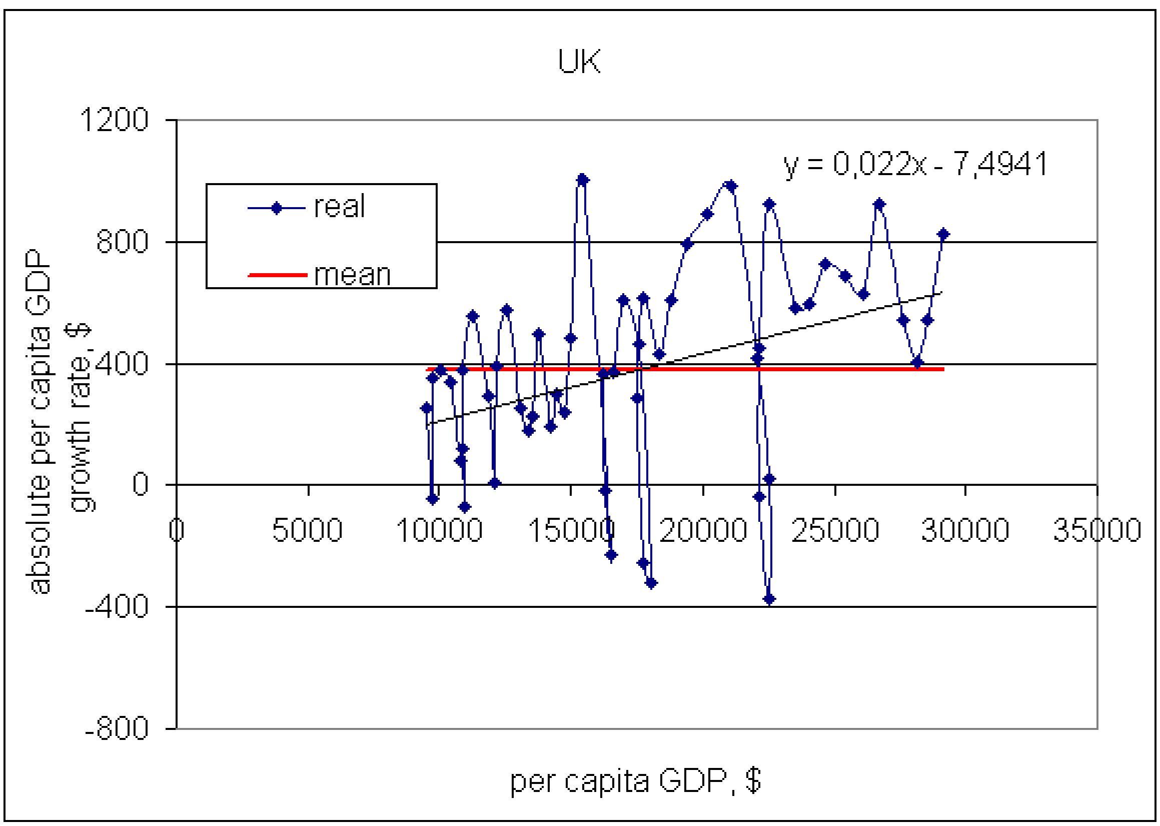


Fig. 25. Comparison of the estimated 18-year-old population in France and the number of 18-year-olds obtained from the real GDP observations according to equation (3).


France
y = -0,0003x + 411,88
real
mean
1200
800
400
0
-400
0
5000
10000
15000
20000
25000
30000
absolute per capita GDP growth rate, $
per capita GDP, $


Italy
y = -0,0034x + 459,47
real
mean
1200
800
400
0
-400
-800
0
5000
10000
15000
20000
25000
30000
absolute per capita GDP growth rate, $
per capita GDP, $


Japan
y = 480
real
mean
absolute per capita GDP growth rate, $
per capita GDP, $
1600
1200
800
400
0
-400
-800
0
5000
10000
15000
20000
25000
30000


UK
y = 0,022x - 7,4941
real
mean
absolute per capita GDP growth rate, $
per capita GDP, $
1200
800
400
0
-400
-800
0
5000
10000
15000
20000
25000
30000
35000

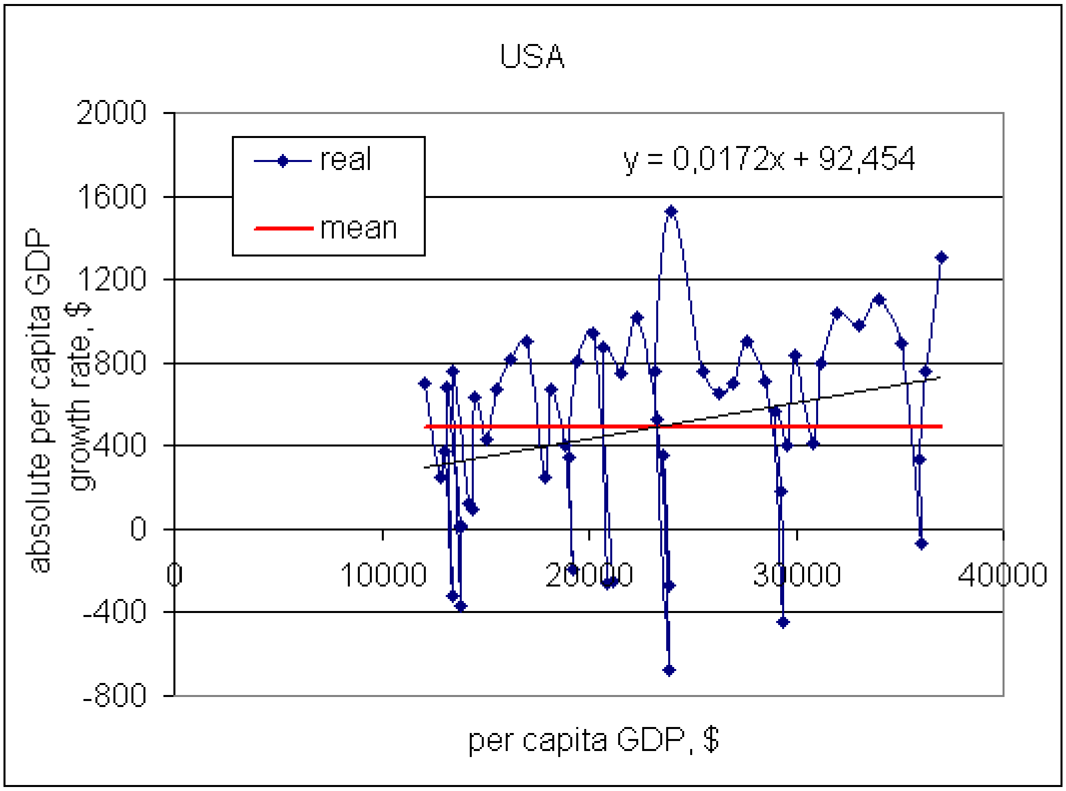


Fig. 26. Approximation of the observed GDP per capita *absolute* growth by a constant for France, Italy , Japan, the UK, and the USA. When applied, linear regression gives the same line for France, Italy, and Japan. For the UK (and the USA), the linear regression indicates some positive growth of the GDP per capita absolute growth. This increase is explained by the effect of the population component change.

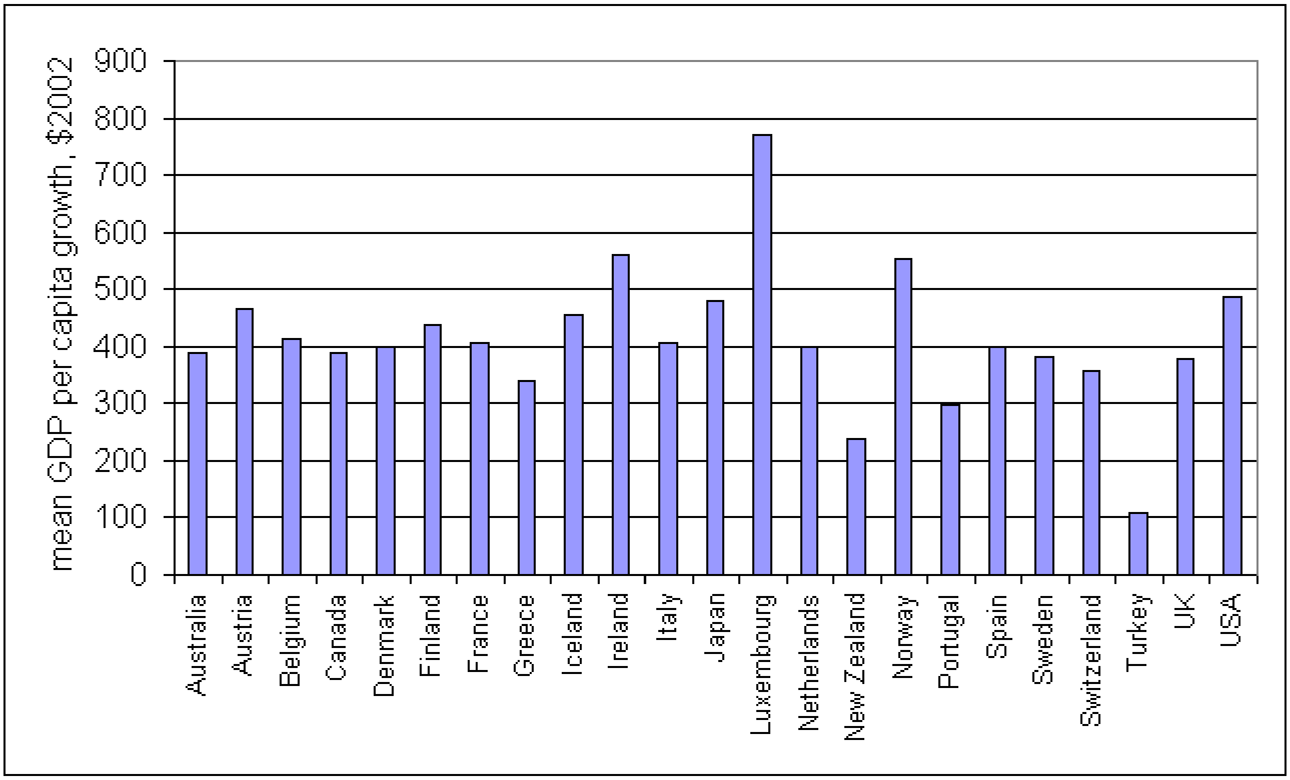


Fig. 27. Mean absolute per capita GDP growth in some developed countries. The values are near $400 per year for most of the countries. Some small economies are characterized by elevated values. When corrected for the population component of growth, the USA has the same value of about $400 per year. Greece and Portugal have been underperforming for a long time in past and New Zealand as well. Turkey is not a developed country and represents developing world in the table. Most of the developing countries are characterized by very low value of absolute per capita GDP growth despite very large relative growth rates. According to the model of economic growth being developed in this paper, these developing countries are not growing at appropriate rate, i.e. the gap with developed counties is monotonically increasing.

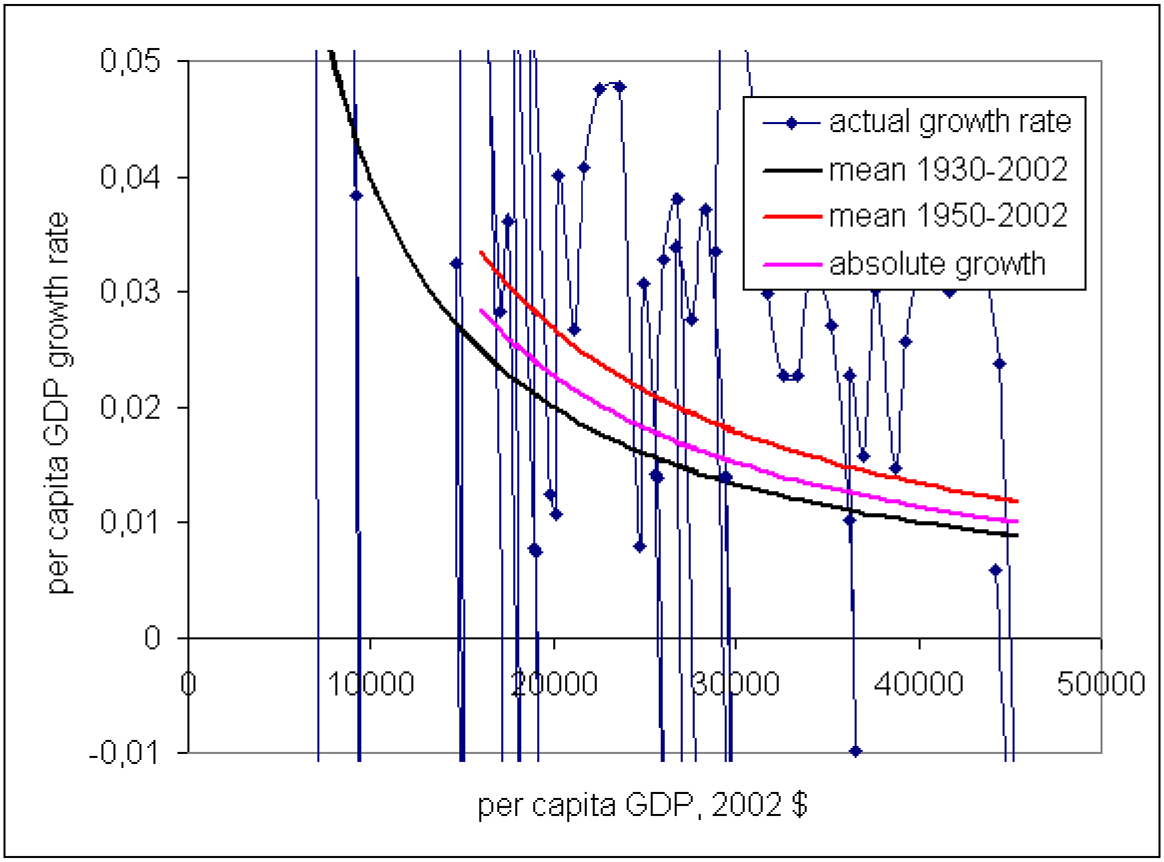


Fig. 28. Approximation (see text for details of the regression procedure) of the observed per capita GDP *relative* growth rate by a function A/G (A is the absolute value of constant growth, G is the GDP per capita) for the USA. Black and red curves correspond to the mean absolute growth values for the period 1930 through 2002 and 1950 through 2002 respectively. The magenta curve illustrates the relative growth related to economic trend only, i.e. the total economic growth less the population component. All the per capita GDP values are corrected for the difference between the total population and the population of 15 years old and above (economically active population). The ratio of the populations in the USA changed from of 1.37 in 1950 to 1.27 in 2002. In 1930, the ratio was 1.41. Thus the per capita GDP values are larger by 1.37 to 1.27 times than in the original table used also for other countries under study.

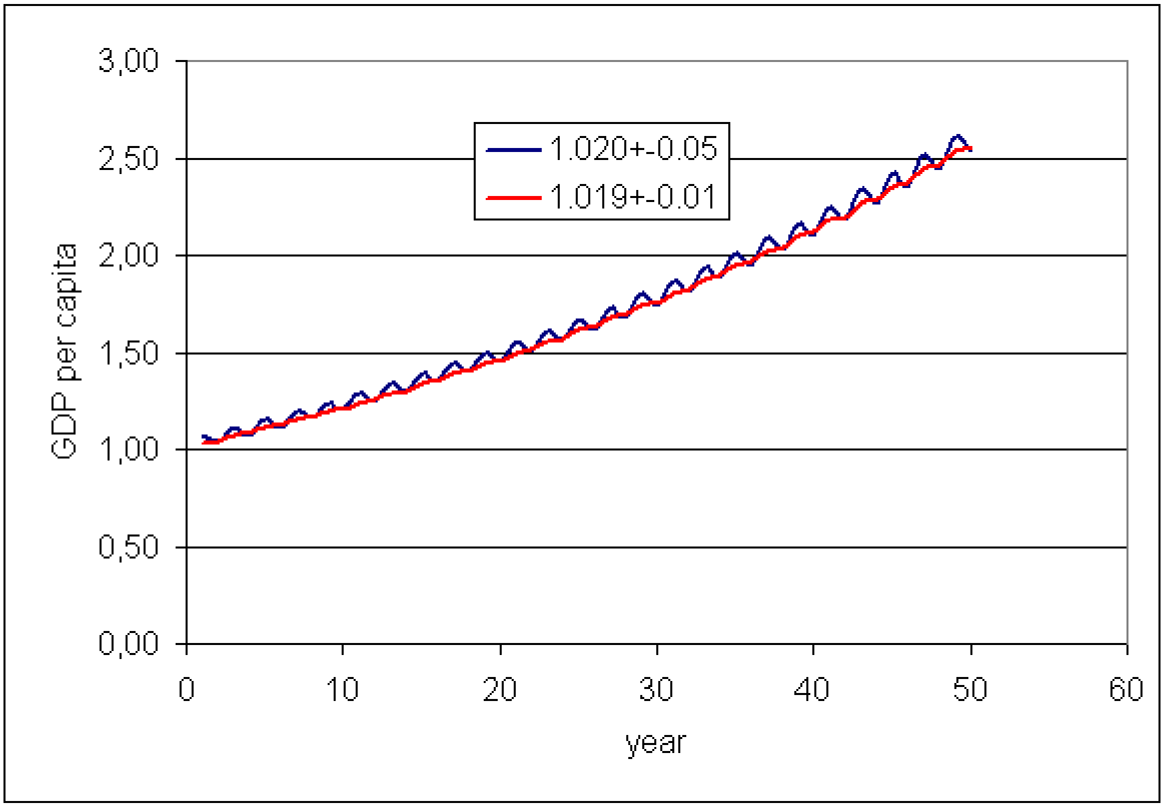


Fig. 29. Illustration of the oscillation amplitude of the growth rate effect on the overall growth. Two curves correspond to the mean growth value of 1.02 with oscillations of 0.05 (i.e. a sequence of growth rates 1.07, 0.97, 1.07 …) and the mean growth value of 1.019 and oscillation amplitude of 0.01. Being formally smaller (1.019^{50}=2.56 < 2.69=1.02^{50}), the latter case provides a larger total growth of 2.56 times compared to 2.53 in the former case. One must not average relative growth rates over long time intervals.

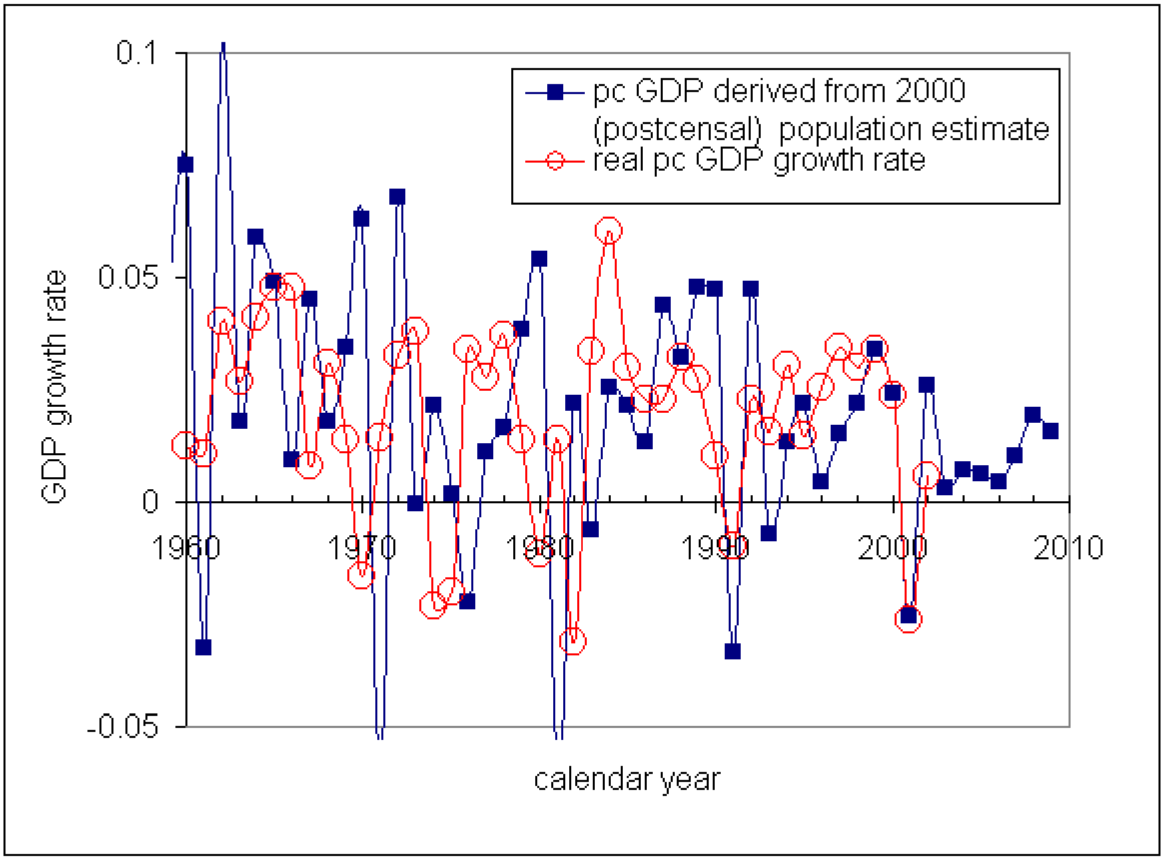


Fig. 30. Predicted by using relationship (7) and observed per capita GDP growth rates in the USA for the period 1960 to 2002.

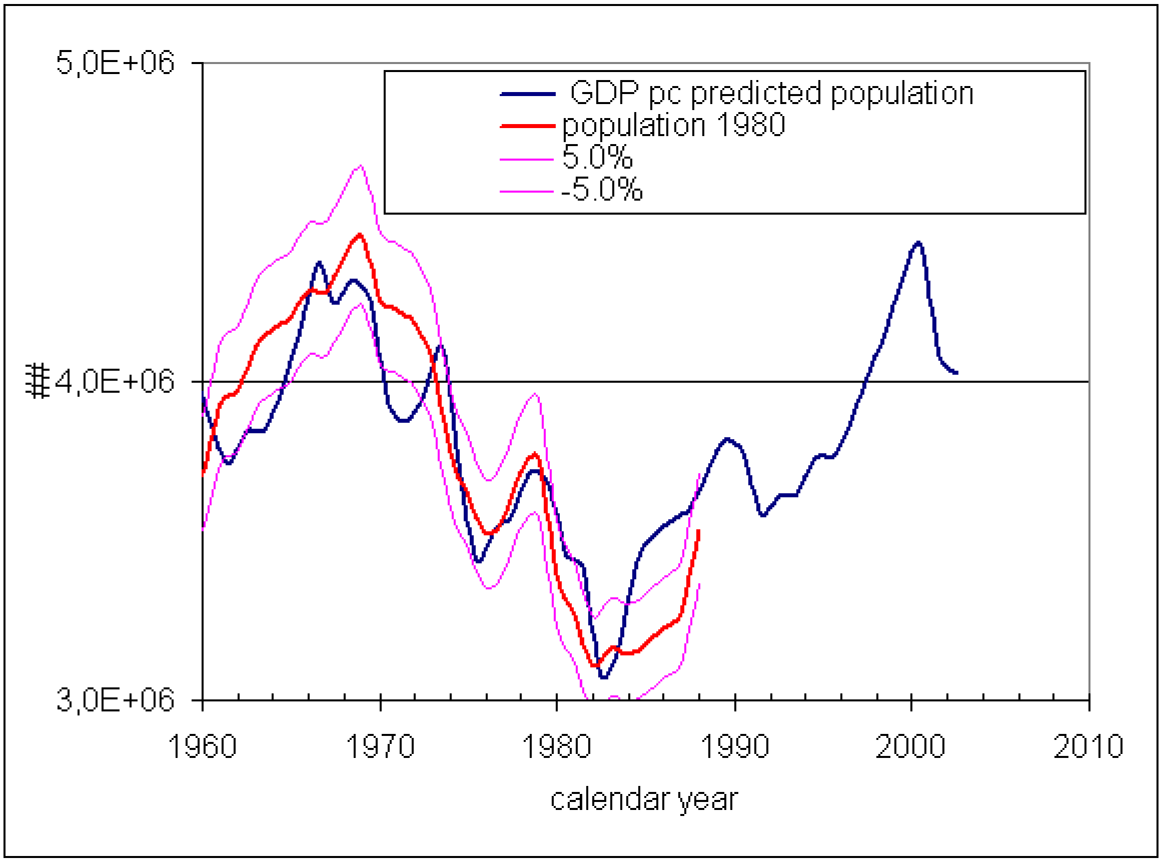


Fig. 31. Comparison of the 9-year-old population as obtained by a back projection from the 1980 population estimate and the number of 9-year-olds obtained from the per capita GDP observations according to equation (8).

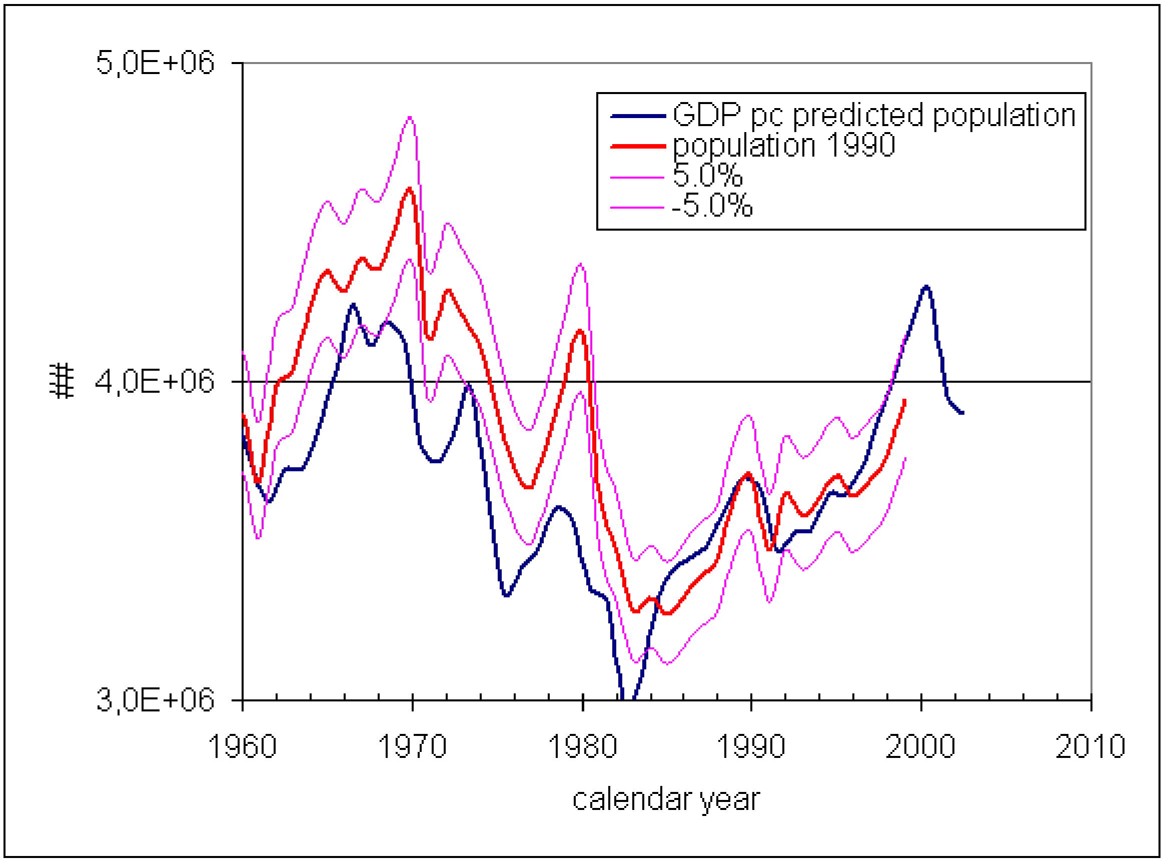


Fig. 32. Comparison of the 9-year-old population as obtained by a back projection from the 1990 population estimate and the number of 9-year-olds obtained from the per capita GDP observations according to equation (8).

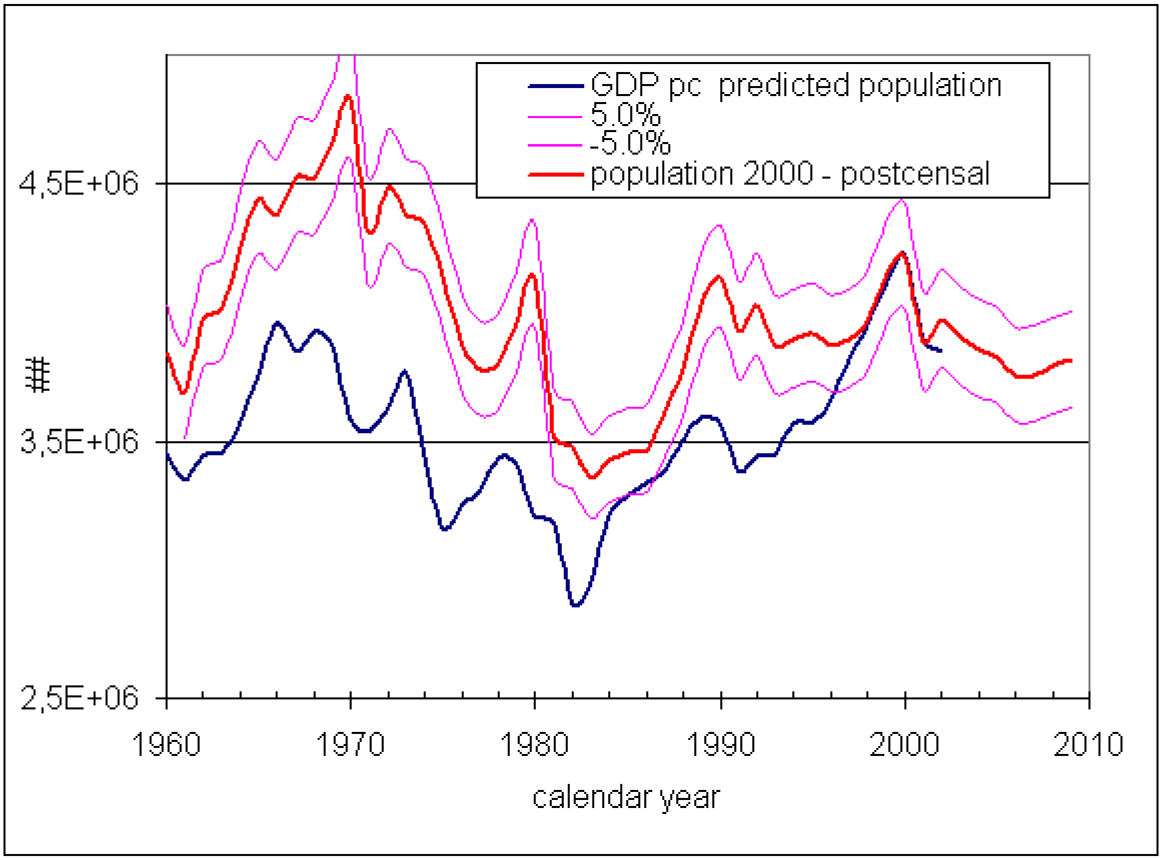


Fig. 33. Comparison of the 9-year-old population as obtained by a back projection from the 2000 postcensal population estimate and the number of 9-year-olds obtained from the per capita GDP observations according to equation (8).

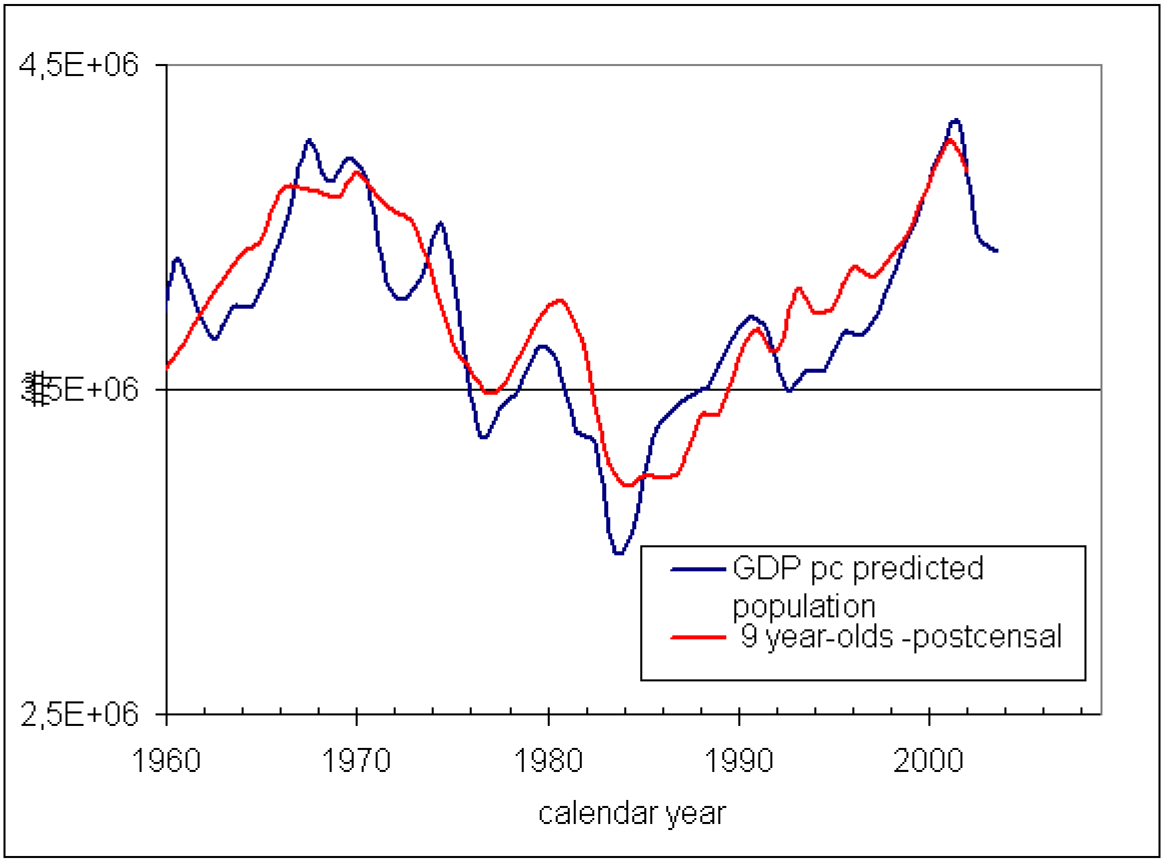


Fig. 34. Comparison of the 9-year-old population (postcensal estimate) and the number of 9-year-olds obtained from the per capita GDP observations according to equation (8). The observed behavior of the 9-year-old population is consistent with the per capita GDP observations.

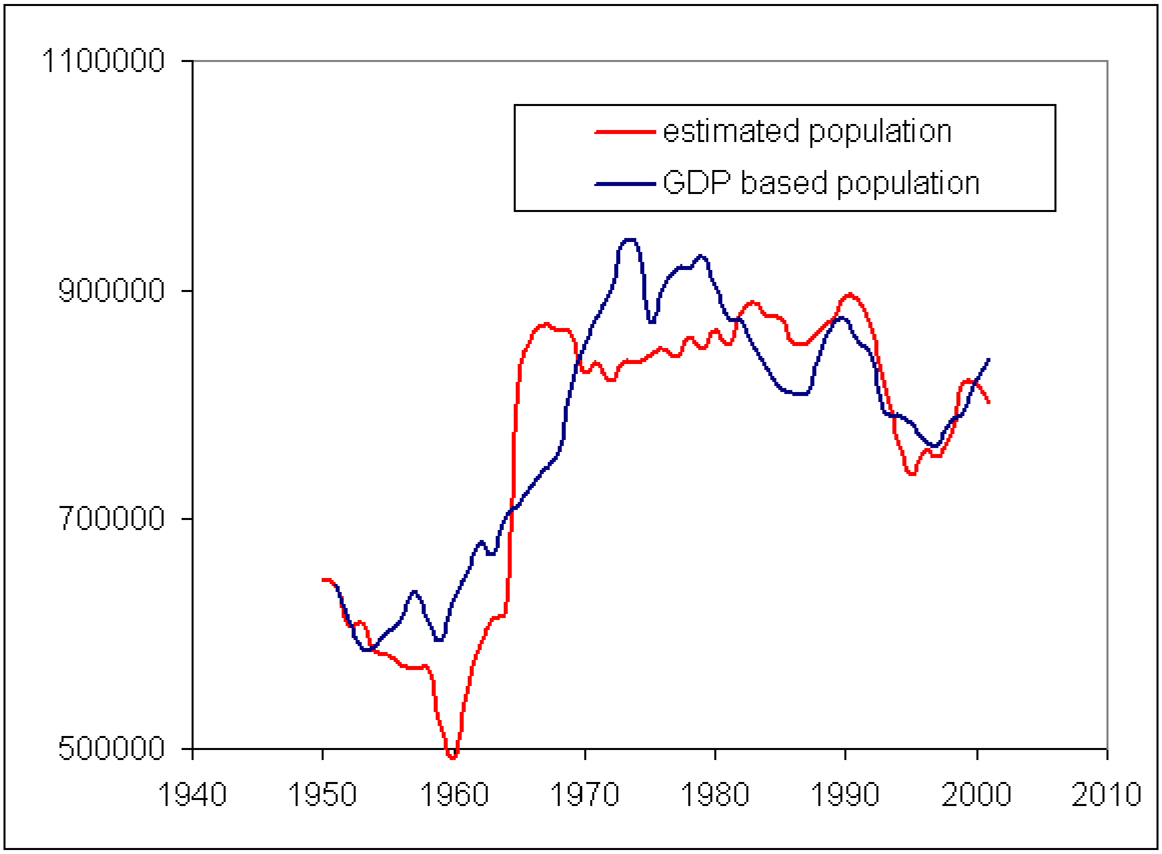


Fig. 35. Comparison of the 18-year-old population in France and the number of 18-year-olds obtained from the per capita GDP observations according to equation (8). The per capita GDP values are corrected for the difference between the total population and population of 15 year s of age and above.